Stochastic Control in Eye Movement Tracking

By

Shean [Wang]

RESEARCH PROJECT

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Electrical Engineering and Computer Sciences

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA, BERKELEY

Approved:

....
Lawrence Stark, Research Advisor

Edwin R Lewis
....
Edwin R. Lewis, Research Advisor

53193982

CONTENTS

range of frequency, the eye can even anticipate the target. This leads numerous models of a predictor for the eye movement system. Consequently, an effort is made to compare the experimental results of this report with those of existing proposed models.

II. The pursuit and saccadic movements of eyes.

Many established experiments have shown that the afferent signals are continuous but the efferent signals are either continuous or discrete or both. Thus the respective eye movement can be pursuit, saccadic, or both pursuit and saccadic. Therefore, questions are raised about the discrete nature of eye movements. Where does the discrete nature come from? How does it work? As of now, there is no physiological explanation. However, from the engineering point of view, there can be numerous possibilities. The most satisfactory model is a stochastic sample-data control system with a uniformly distributed inter-sampling time. This system will be reviewed in this report.

EXPERIMENTAL METHOD

The experimental procedure for the measurement of horizontal eye movements has been well established and therefore only a brief description will be presented here.

In a dark room, the subject was asked to fixate on a light spot which is controlled by a target motion generator. A small beam of infra-rad light was directed to the left eye ball, and its reflection was collected by a pair of photocells aimed at the iris-scleral border as shown in Figure 3a. Eye positions could be detected by the difference in output of these two photocells. Then an operational amplifier was used to amplify the difference and send it to a Sanborn recorder. In order to minimize the interference introduced by motion, the subject leaned his forehead against a firm bar and bit on a piece of dental impression wax as shown in Figure 3b.

Experimental data were collected from three subjects. Three different kinds of stimuli were presented: regular square wave, irregular square wave (or staircase wave), and ramp input. The latency of response to each stimulus could be defined in three different ways as shown in Figure 4a, 4b, and 4c. However, in Figure 4c, it was very hard to decide to which particular input the eye did respond. The time required for eye to respond upon a stimulus (reaction time) was about 150 msec, therefore, the time larger than but closest to 150 msec was choosen.

For each experimental run, a histogram of latency of response was generated. The binwidth was 25 msec, and the middle point of each bin was used as the latency within its

± 12 msec range (see Figure 5 to Figure 11). The average and standard deviation of latency were computed by IBM 6400 for each experimental run of both square wave and staircase wave stimuli. A set of experimental data was collected from the same subject with the same stimulus by changing the frequency alone. The frequencies used are listed in Table 1 (see next page). For each set of square wave and staircase wave experimental runs, the median(M), average(Av), mode(Md), and standard deviation(s) of latency were ploted vs. frequency as shown in Figure 5 to Figure 10. In the ramp input case, only median vs. frequency was ploted.

Set I. Regular square wave input:

1. 0.1 Hz.
2. 0.3 Hz.
3. 0.5 Hz.
4. 0.9 Hz.
5. 1.1 Hz.
6. 1.3 Hz.
7. 1.5 Hz.
8. 1.7 Hz.
9. 1.9 Hz.
10. 2.3 Hz.

Set II. Irregular square wave inputs:

1. 0.130 Hz, 0.235 Hz, 0.488 Hz.
2. 0.235 Hz, 0.488 Hz, 0.785 Hz.
3. 0.488 Hz, 0.785 Hz, 1.230 Hz.
4. 0.785 Hz, 1.230 Hz, 1.480 Hz.
5. 1.230 Hz, 1.480 Hz, 1.710 Hz.
6. 1.480 Hz, 1.710 Hz, 1.950 Hz.
7. 1.710 Hz, 1.950 Hz, 2.210 Hz.
8. 1.950 Hz, 2.210 Hz, 2.450 Hz.
9. 2.210 Hz, 2.450 Hz, 2.730 Hz.
10. 2.450 Hz, 2.730 Hz, 3.010 Hz.

Set III. Ramp inputs:

1. 2°/sec.
2. 1°/sec.
3. 0.7°/sec.
4. 0.833°/sec.
5. 0.5°/sec.
6. 0.33°/sec.
7. 0.133°/sec.
8. 1.6°/sec.

Table 1.

OBSERVATION FROM THE RESULTS

(A) Regular square wave inputs:

The results from subject C (Figure 5) and subject W (Figure 7) give similar conclusion for the values of the median, the average, and the mode of the latency distribution curve. They show that these three values are all frequency dependent. But at the meantime, subject C and subject S show a similarity in the curve of the standard deviation of latency distribution vs. frequency. That is, the standard deviation of the latency distribution is also frequency dependent.

For the latency distribution curve,

Let M = median

Av = average

Md = mode

s = standard deviation

then the above similarities can be summarized as follows:

(1) $0 \leq$ frequency < 0.5 Hz.

a. M, Av, Md are all decreasing as frequency increasing.

b. s is increasing as frequency increasing.

(2) 0.5 Hz. $\leq$ frequency ≤ 1.5 Hz.

a. M, Av, and Md are within $\pm$ 25 msec range.

b. s is decreasing as frequency increasing.

(3) frequency > 1.5 Hz.

M, Av, Md, and s are increasing as frequency increasing.

Therefore the distribution of latency is shifted as frequency is changed. The distribution starts at the large time lag region, then shifts to the time lead region. As

frequency continues to increase, it shifts back towards the time region. Subject S (Figure 6) yields a set of different results for median, average, and mode. These three values are all between 100 msec and 175 msec. The distribution curves do not have any obvious shift as frequency changed.

The standard deviation curve from subject W shows that the value varies between $\pm$ 35 msec to $\pm$ 88 msec, and it is frequency independent.

Despite the differences among the above discussed three subjects, they do have some similarities. When frequency was increased, the number of abnormal responses of all three subjects would increase, and their distribution curves all were normal form.

(B) Irregular square wave inputs (staircase wave) (Figure 8 to Figure 10) :

The median, average, mode, and standard deviation of the latency distribution are not all frequency dependent. The values of median, average, and mode varied within the range of 150 msec to 250 msec. But the shape of the latency distribution curve seems to be frequency dependent. As frequency increases the distribution curve tends to change its shape from a normal distribution to a square form distribution. This change can be observed by comparing Figure 8a to 8c Figure 9a to 9c, and Figure 10a to 10c.

(C) Ramp inputs (Figure 11):

(1) The speed of target 0.33°/sec:
Latency decreased as speed increased.

(2) 0.33°/sec the speed of target 0.83°/sec:
As speed increased, the latency of response to the continuous portion increased (Figure 11), and the

latency of response to the discontinuous portion decreased (Figure 11).

(3) The speed of the target 0.83°/sec:
Latency of response decreased as speed decreased.

(4) The latency of response to the discontinuous portion is much less than the latency of response to the continuous portion as shown in Figure 11d.

DISCUSSION

In this report, the discussion was divided as two parts: one emphasizes the pattern recognition associated with the eye movement, the other one concentrates on the discrete nature of eye movement. In either case, different models were discussed. Comparison was made between models or between those models and my experimental results.

There were three models discussed in the pattern recognition phenomenon, namely, Dallos and Jones' model, Sugie's model Cyr and Fender's model. In the discrete phenomenon, Forster's model was reviewed, and Latour's experimental results were mentioned.

(A) Pattern Recognition

(1) Dallos and Jones' model:

From the above presentation, it is obvious that the distribution of latency is very much dependent upon both the wave form and the frequency of the stimulus. In fact, this is the property that led Dallos and Jones[1] to theorize about the "acquisition of learning". From their theory, a model is proposed. Three properties of this model are worth discussing, namely, learning speed, prediction, and detection.

(a) Learning Speed

By examining the first few cycles of tracking responses to sinusoidal and square wave inputs (Figure 12), Dallos and Jones believed that the rate of learning was considerably greater for continuous than for discontinuous input with the same frequency. But from my experimental

observation (see previous section and Figure 11d), the response of the ramp tracking showed that it is not so. When both continuous and discontinuous changes were presented in a repetitive manner, namely the ramp stimulus, the rate of learning in the discontinuous portion was faster than that of continuous portion (Figure 11d), thus contradicted their conclusion. Therefore, the correlation, if any, between the rate of learning and the continuity of stimuli remains still a question.

(b) Predictor

For each input stimulus, an output Bode plot was ploted. Then the corresponding closed-loop transfer function was obtained by curve fitting. Basing on the assumption of linearity and unit feedback, the corresponding open-loop transfer function for each stimulus can be computed from its closed-loop transfer function.

Let $G_p(jw)$ = closed-loop transfer function for sinusoidal input.

$G(jw)$ = closed-loop transfer function for random input.

$g_p(jw)$ = open-loop transfer function for sinusoidal input.

$g(jw)$ = open-loop transfer function for random input.

then $g_p(jw)$ and $g(jw)$ can be computed from the following equations:

$$G_p(jw) = \frac{g_p(jw)}{1+g_p(jw)}$$

$$G(jw) = \frac{g(jw)}{1+g(jw)}$$

By comparing the two open-loop transfer functions, $g_p(jw)$ and $g(jw)$, a hypothetical predictor transfer function $P(jw)$ was defined as follows:

$$P(jw) = \frac{g_p(jw)}{g(jw)}$$

Since this predictor was derived mathematically, its validity should be examined by various ways.

Firstly, how reliable is this predictor? According to Dallos and Jones, the predictor should give a phase lead starting from 100 degrees increasing to 250 degrees as frequency increased from 0.5 cps to 2.5 cps. In our experiment, the subject S was presented with square wave stimuli (as shown in Figure 6d). The frequency distribution of latency for the subject showed that the existence of prediction is questionable. As frequency changed, there was no significant shifts in the distribution curve. (The latency of the response fall into the interval of 100 msec and 150 msec) It therefore implied that the response of subject S failed to predict any of the stimuli within the applied range of the frequency of the square wave.

Secondly, under a specific frequency, what kind of distribution curve of latency would the predictor have? Because by examining this curve, I believe that the efficiency of the predictor could be obtained. A sharp and narrow curve (eg. a normal distribution) could represent an effective predictor. On the other hand, a flat and wider curve (like rectangular shape) could mean a weaker one. The efficiency of the proposed predictor was not discussed.

Thirdly, since learning was an unconventional process, it should not be inconceivable that the predictor might not be

By comparing the two open-loop transfer functions, $g_p(jw)$ and $g(jw)$, a hypothetical predictor transfer function $P(jw)$ was defined as follows:

$$P(jw)=\frac{g_p(jw)}{g(jw)}$$

Since this predictor was derived mathematically, its validity should be examined by various ways.

Firstly, how reliable is this predictor? According to Dallos and Jones, the predictor should give a phase lead starting from 400 degrees decreasing to 250 degrees as frequency increased from 0.5 cps to 2.5 cps. In our experiment, the subject S was presented with square wave stimuli (as shown in Figure 6d). The frequency distribution of latency for the subject showed that the existence of prediction is questionable. As frequency changed, there was no significant shifts in the distribution curve. (The latency of the response fall into the interval of 100 msec and 450 msec) It therefore implied that the response of subject S failed to predict any of the stimuli within the applied range of the frequency of the square wave.

Secondly, under a specific frequency, what kind of distribution curve of latency would the predictor have? Because by examining this curve, I believe that the efficiency of the predictor could be obtained. A sharp and narrow curve (eg. a normal distribution) could represent an effective predictor. On the other hand, a flat and wider curve (like rectangular shape) would mean a weaker one. The efficiency of the proposed predictor was not discussed.

Thirdly, since learning was an unconventional process, it should not be inconceivable that the predictor might not be

physically realized. But from a physiological point of view, it would be an interesting and meaningful thing to know the possible mechanism or location of the predictor. But Dallos and Jones made no attemp to suggest it.

(c) Detector

Since the operation of the predictor depended upon the wave form of stimulus, there should be another element, namely, detector, which could exhibit the predictor under periodic stimuli and could inhibit it under aperiodic stimuli. A simple detector was suggested by Dallos and Jones. The detector included a memory unit M with a finite decay time, and a comparator D (Figure 12). The comparator would continuously compare the output signal from the memory unit and the present error signal. If the two signals were dislike, then the error signal would bypass the predictor via a direct path toward the plant, as the cases for aperiodic stimuli. If the two signal were alike, then the error information would be channelled through the predictor.

Based on this model, Dallos and Jones tried to explain the frequency response of square wave stimuli. There were three cases:

1. For low frequency stimuli:
   Since the memorizable information for a square wave appeared to be the time duration between the transitions, therefore, when the time duration was longer than the memory trace, then no information could be stored, consequently, no learning process could proceed.
2. For high frequency stimuli:
   Since the target motion was too fast, even with a periodic input, the error signal could not be a periodic one. So learning failed.
3. For intermediate frequency stimuli:
   A steady state tracking could well be established. The

amplitude of distortion in eye motion was not significant. Within this range of frequency, there was almost complete synchronism between input and output, therefore, the memory trace could not be continuously reinforced. Since the memory trace was assumed to decay with time, then after some period of time, the loss of synchronism should be expected. Dallos and Jones observed that in the steady state tracking, the average time duration for good tracking period was approximately 5 to 7 seconds. Following those good tracking periods, there were a few cycles without compensation.

From the above explanation of the square wave response, it seemed to me that there were some points open for argument:

(a) It is questionable, that whether the large value of latency during a high frequency target tracking was caused by the aperiodic error signal or not. If we took eye muscle into consideration, it should be easy to demonstrate that the eye could not move as fast as one wished to. Suppose a subject was asked to count repeatedly "one, two, one, two, one, ---", and simultaneously, the subject's eye moved back and forth between two points in front of him, the counting of number and the movement of the eye were synchronized. If the counting started at a low frequency, the eye could keep the synchronization very well. If the speed of counting was increased, the eye gradually become unable to follow. The back and forth movement of the eye would get worse and worse as the speed of counting increased. In this demonstration, the brain knew the speed and the exact positions of the two fixation points. The speed and position were not known by "learning" but by "given". the failure of the eye showed that there

was a speed limitation for the eye. It should not be confused with the arbitrary eye movement, namely, the eye was asked to move back and forth between left and right without specific position to fixate. In this case, because no accuracy was required, the eye could respond faster than in the former case.

In the case of eye tracking movement, the speed of the eye and the positions for fixation were not given. If the eye could "learn", as suggested by many models, it should be expected that the response would not be better than the "given" case. Thus the bad tracking response of a high frequency square wave stimulus could be caused simply by the limitation of the eye muscle movement. I was one of the subjects. According to my own experience, at high frequency tracking, I could "feel" or "sense" the frequency of the target, even when my eye failed to follow it. Therefore, it seemed to me that the "learning behavior" should belong to the brain, not to the eye. Thus when the target frequency was high, and the eye failed to track properly, the brain still could "learn" the frequency of the target, if the target motion was periodic.

(b) There were many mechanical models for the muscle. No matter which one was the best, at least they implied that the muscle possessed mechanical properties. Therefore, it would be proper to assume that there was a nature frequency for the eye muscle mechanism. If the frequency of the target motion was close to the natural frequency of the eye muscle mechanism, then a resonance phenomenon might happen. This was probably the reason for the good tracking responses for the intermediate frequency stimuli.

The above argument was just induced from "common sense" and "experimental feeling", no strong evidence or experimental results supported it. However, it seemed to me that using mechanical (or mathematical) model of physiological properties to approach the eye tracking system might be more realistic and practical than a pure mathematical derivation or curve fitting.

(2) Sugie's model:

A different approach to the study of the "predictive control" of the eye movement was carried out by Sugie. After studying the response to regular square wave inputs, he proposed a mathematical model of estimation based on stochastic optimal control concept (Figure 14 and Figure 15). The variable to be estimated was the period of the target motion, and the statistical parameter was the variance of the estimated period.

Four assumptions were made. (i) The estimation depended upon the target frequency and the number of target cycles. (ii) The estimated period of the target motion was accompanied with some stochastic randomness caused by the uncertainty of human memory. (iii) The mean of the estimated period coincided with the period of the target motion, because the probability function had no reason to be biased. (iv) The variance of the estimated period should depend on the target frequency and the number of target cycles (Figure 16). A brief reason for the last assumption was worth mentioning:

(a) The dependence of the variance upon target frequency: As Dallos and Jones suggested, for a square wave input, the useful information could be obtained only twice a cycle. When frequency was low, the memory decay would cause uncertain information, therefore, the variance of the estimated period could be assumed

large. However, when the target frequency was very high, the sample data phenomenon was taking into account. Accourding to L. R. Young and L. Stark[5], the sampling period was 250 msec. Therefore, when the input period became compatible with the sampling period, the change of the target position could not be sensed very accurately. Consequently, the variance of the estimated period could be assumed large. Thus the variance should have its minimum value at the intermediate input frequency range.

(b) The dependence of the variance upon the number of cycles of target motion:
As more information became available, the variance of the estimated period should be decreased. Consequently, the variance of estimated period should be a monotonically decreasing function of the number of cycles.

For simplification, he assumed a rectangular probability density function (Figure 18), that is,

$$\text{probability density} \begin{cases} = \frac{1}{2}\, y' & -y' \leq t \leq y' \\ = 0 & \text{otherwise} \end{cases}$$

The estimation was assumed to optimize a "performance index". This performance index PI was defined as a mean-squared value of the length of time by which a stimulus preceded or laged behind a response.

$$PI = \int_{-(y'-t_{opt})}^{x} t^2 \frac{1}{2y'}\, dt + x^2 \frac{(y' + t_{opt} - x)}{2y'}$$

where x = constant reaction time without prediction.
t_{opt} = optimal reaction time.

The response of this model (Figure 17) gave a good suggestion for the possible range of the mode and average for the distribution of latency. But it failed to give a desired distribution curve.

As Dallos and Jones, Sugie assumed the memory decayed as the only explanation for the low frequency square wave response. However, he had a different explanation for the higher frequency response. He considered the discrete nature of the saccadic eye movement which, I believe, was more realistic than the explanation given by Dallos and Jonse.

Sugie assumed that the variance also depended upon the cycle number of the target motion, that is, the variance was a monotonically decreasing function of cycle number, independent of target frequency. Statistically, it should be true. But practically, it would be doubltful. When conducting experiments of a longer time duration, the eye might be fatigue, which could give unreliable responses. Therefore the variance could be larger as more experimental runs were recorded. Tn the derivation of his model, Sugie assumed the number of cycles of the target motion to be infinite. Since the data he used was from several other models, therefore the cycle numbers would not necessary be equal nor infinite. It seemed to me that the dependence of the variance upon the cycle number was not a proper assumption.

Another interesting assumption worth mentioning was concerning the unbiased probability density function. A rectangular function was assumed for simplification. But Sugie did not suggest any other suitable functions. Whether a normal distribution function could give a desirable distribution curve for latency or not was worth to know.

(3) Cyr and Fender's model:

After a number of predictive models (Fender and Nye, 1961; Young and Stark, 1963[5], Robinson, 1965) for the eye tracking system were proposed, the existence of the predictive ability of the eye movement was challenged by Cyr and Fender. Based on the study of human eye movement in two dimensional tracking task, Cyr and Fender found that the system was non-linear. Therefore, a transfer function could be derived for the oculomotor system. Thus it should not be possible to predict the response to one class of target motion by linear combinations of the responses to other classes of stimuli (as in the Dallos and Jon model[1]) Consequently, the computation of a minimum phase lag (Dallos and Jones, 1963[1]; Young and Stark, 1963[5]) should not be possible. Accordingly, the latency was only a simple delay which depended on the class of target motion.

Since all of my experimental results of this report were from one dimensional eye tracking movement, therefore no comparision between Cyr and Fender's results and mine could be carried out. However, there was a point which might be worth mentioning. In the one-dimensional eye tracking movement, the ability to predict was based not only on the frequency of the stimulus, but also on the shape of the target motion. In other words, the eye could predict, if the stimulus was of periodic form with a intermediate frequency. Thus for disproving the predictive two-dimensional input with a intermediate frequency should be used. But Cyr and Fender only used two kinds of stimuli, one was random signal, the other one was a sum of four small sinusoids in both vertical and horizontal directions. Therefore, no periodical input was used by Cyr and Fender. Consequently, their conclusion was not strong enough.

(B) Discrete Nature Of Eye Movement

The discrete nature of the saccadic eye movement control is well established. Since the introduction of the sample-data model by Young and Stark[5], the analysis of this discrete nature has developed numerous refined models (see reference 6, 7, 8). There are three different types of sample-data systems. A model has either target-synchronized or target-asynchronized properties. A target-synchronized system is c ock-asynchronized. But a target-asynchronized system can be either clock-synchronized or clock-asynchronized. These definitions for the three types of sample-data systems are summarized in Table 2.

	closk-synchronized	clock-asynchronized
target-synchronized	impossible	possible
target-asynchronized	possible	possible

Table 2

(1) Latour's experimental results:

In the early studies of horizontal eye movement by Latour and Bouman[9], the frequency distribution of reaction time for both short run (Figure 24) and long run (Figure 25) were obtained. Latour noticed that, in short sessions the reaction time distribution was multimodal, such that:

$$t = \bar{t} \pm ka$$

where t = reaction time

$\bar{t}$ = most frequent t value

k = 0, 1, 2, 3, - - - -

a = 20 to 40 msec.

In longer sessions, this phenomenon faded out. But the distribution of the "difference between the reaction time and its succeeding movements" showed a similar frequency distribution curve.

$$\text{Let } \Delta t = t_n - t_{n+1}$$

where t_n = reaction time at the nth reaction

t_{n+1} = reaction time at the (n+1)th reaction

Δt = difference between the reaction time and its succeeding movement.

then the distribution of Δt (Figure 26) could be represented by

$$t = \bar{\tau} \pm ka$$

where $\bar{\tau}$ = some positive value

By comparing the frequency distribution of reaction time t and the frequency distribution of the Δt (Figure 26), Latour concluded that there was a continuous decrease in the " most

frequent reaction time $\bar{t}''$, followed by a relatively fast increase of $\bar{t}$ within a period of about 20 seconds (Figure 27). In other words, for a long session tracking, the eye could graduatly shorten its reaction time then rapidly drift back to the starting value of reaction time. This cycle could be continued as the experiment went on.

Latour's experimental results and observations could be in the discussion of the following Forster's model.

(2) Forster's stochastic sample-data model:
The studies by Lang[7] showed that the model for the discrete nature of the eye movement was more satisfactory if it was randomly sampled. Forster[8] reexamined the three possible types of interesample time distribution function (see Table 2, page 19) with his revised sample data model (Figure 19).

Started from target--asynchronized system, he assumed that:

$$L = t_1 + D$$

where L = latency

D = dealy time from occurrence of the input to the next occurrence of a sampling instant (-from this definition D should be strictly non-negative).

t_1 = constant delay time, as in Figure 20.

$f_x(x)$ = probability density function of the variable x (any of the above variables).

Since D was a random variable, L should also be a random variable, thus the density function would be related by:

$$f_L(L) = f_D(D + t_1)$$

Based on the assumptions that:

(a) the input occured in as interval τ_c,

(b) the time between inputs were greater than τ_{max}, the maximum value of τ_c,

(c) the input was uniformly distributed

Forster derived out $f_D(D)$:

$$f_D = \begin{cases} \dfrac{1}{\tau_{max}} \displaystyle\int_D^{\tau_{max}} f_\tau(\tau)\, d\tau, & \text{for } D \geq 0 \\ 0 & \text{for } D < 0 \end{cases}$$

Since $f_D(D)$ was a non-increasing function, thus it should give a non-increasing latency histogram (Figure 21). But the actual latency distribution increased for low latencies. Therefore, Forster suggested that for asynchronized sample, the eye sometimes was able to shorten the sampling intervals. when a step was observed. This hypothesis could be supported if the function was as follewed:

$$f_\tau(\tau) = \begin{cases} 0.0133 & 60 \leq \tau < 120 \\ 0.0025 & 120 \leq \tau \leq 200 \\ 0 & \text{otherwise} \end{cases}$$

The corresponding simulation latency histogram from this function was showen in Figure 22. After comparing the simulations and the experimental results, Forster showed that the eye control system had characteristics of both target-synchronized and target-asynchronized. But the best sampler logic should be non-synchonized with a sample T which was composed by a constant T_o plus a noise n, i.e.

$$T = T_o + n .$$

Examining Forster's model by Latour's conclusion and my experimental results, two points should discussed:

(a) The sampler logic of Forster's model was uniformly distributed in the interval of 150 to 250 mseconds. When the period of a square wave stimulus was compatible with the sampling interval, then the change of target position could not be accurately sensed or sometimes even missed (Figure 23). Thus as increased percentage of abnormal responses should be expected as the frequency of the input increased. Conversely, for a low frequency stimulus, the period of the input was much larger than the interval of sampler, thus a small value of percentage of abnormal response was able to be obtained. According to my experimental results shown in Figure 5 to Figure 7, the above was confirmed.

(b) In the assumption of Forster's model, the latency was defined as:

$$L = D + t_1$$

where t_1 was constant delay time, and D was the time interval between the occurence of input and the occurance of the next sampling instant (Figure 20). It should be obvious that latency L was proportional to the value of D. However, considering Latour's difference phenomenon, the mode of the latency distribution curve tend to decrease as the time for experimental run increased (Figure 27). It seemed to me that the decrease of the mode of latency should imply the decrease of latency. Thus the value of D should also be decreaseing. The decreasing of D could only be achieved by shortening the sampling period. If this was true, then the eye could sense the change of target position more accurately. Thus, for a highfrequency stimulus, the percentage of abnormal response duringa long session run could be reduced. Since all of my experimental runs were

POSSIBLE FUTURE DIRECTION

Based on the review and examination of those models disscused in the above section, some possible future experiments could be conducted:

(1) For a furthur understanding of "learning behavior" and "learning speed", a detailed comparison of the difference between the response of continuous stimuli and discontinuous stimuli could be done. Therefore, more experiments could be carried out in this direction.

(2) As far as learning behavior as concerned, the two-dimensional eye tracking movement should be tacken into account. Cyr and Fender only used two types of stimuli: one was a random signal, the other one was a sum of small sinusoids in both vertical and horizontal direction. Since the one dimensional eye tracking system showed that the latency distribution curve was both input frequency and input shape dependent, and furthur more, it was believed that in some models a periodic input with proper frequency could be predicted by the eye, thus a two-dimensional periodic input should be used in order to make a comparison with the one dimensional results. A simple two-dimensional stimulus could be made by a square wave movement in both vertical and horizontal direction. The response of this input might bring some information such as whether or not the latency distribution curve would shift to some phase lead region as the input frequency varied to a certain range. No matter what kind of results could be obtained, it should be halpful for the furthur analysis of the "learning behavior".

(3) The oversimplification of Sugie's probability density

function failed to give a proper latency distribution curve. Since the latency distribution curve was similar to normal distribution, therefore, a probability density function similar to normal distribution might be reasonable to try.

(4) Longer session experimental runs could be carried out. The results could be used for the following purpose:
   (a) Re-examine Latour's difference phenomenon.
   (b) Re-check Dallos and Jones' memory decay phenomenon, especially the periodic forgetness in a steady state tracking movement.
   (c) Whether the percentage of abnormal response would be decreased or not, as suggested in the discussion of this report.
   (d) It might be useful for a furthur study of Forster's sample-data model.

SUMMARY

One dimensional eye tracking movement was measured. Following types of stimili were presented, namely, regular square wave, irregular square wave (or staircase wave), and ramp input. The results showed that latency distribution curve was frequency dependent. For square wave, the curve similar to a normal distribution shifted back and forth between the time lag region and the time lead region. For staircase wave, the cruve changed its shape from a normal distribution to a regutangular distribution as frequency increased. The response of ramp inputs showed that when both continuous and discontinuous motion were involved, the discontinuous portion was faster than the continuous portion.

Based on our experimental results, the strong and weak points of some proposed models were discussed.

(1) Dallos and Jones' model:
   (a) Strong point: It could explain the response for square wave stimili.
   (b) Weak point:
      i. The dependence of learning speed on the continuity of input was questionable.
      ii. The predictor was purely derived out from curve fitting and mathematic equations. There was no simulation results. The efficiency of the predictor was discussed. The existance of this predictor was doubtful.

(2) Sugie's model:
   (a) Strong point:
   For square wave stimuli, the model could give a good suggestion of the possible range of the mode and the average of the latency distribution curve.
   (b) Weak point:

SUMMARY

One dimensional eye tracking movement was measured. Following types of stimuli were presented, namely, regular square wave, irregular square wave (or staircase wave), and ramp input. The results showed that latency distribution curve was frequency dependent. For square wave, the curve similar to a normal distribution shifted back and forth between the time lag region and the time lead region. For staircase wave, the curve changed its shape from a normal distribution to a rectangular distribution as frequency increased. The response of ramp inputs showed that when both continuous and discontinuous motion were involved, the discontinuous portion was faster than the continuous portion.

Based on our experimental results, the strong and weak points of some proposed models were discussed.

(1) Dallos and Jones' model:

(a) Strong point: It could explain the response for square wave stimuli.

(b) Weak point:

i. The dependence of learning speed on the continuity of input was questionable.

ii. The predictor was purely derived out from curve fitting and mathematic equations. There was no simulation results. The efficiency of the predictor was discussed. The existence of this predictor was doubtful.

(2) Sagie's model:

(a) Strong point:

For square wave stimuli, the model could give a good suggestion of the possible range of the mode and the average of the latency distribution curve.

(b) Weak point:

i. The dependence of variance of latency distribution upon the cycle number of the target motion was questionable.

ii. Oversimplification of the probability density function.

iii. Failed to predict the shape of latency distribution curve.

(3) Cyr and Fender's model:
This model was not discussed here. It was only a reference to show a different point view. However, it could be safe to say that the two-dimensional tracking was a good approach. But the type of stimuli should be one which was comprable to the one-dimensional tracking.

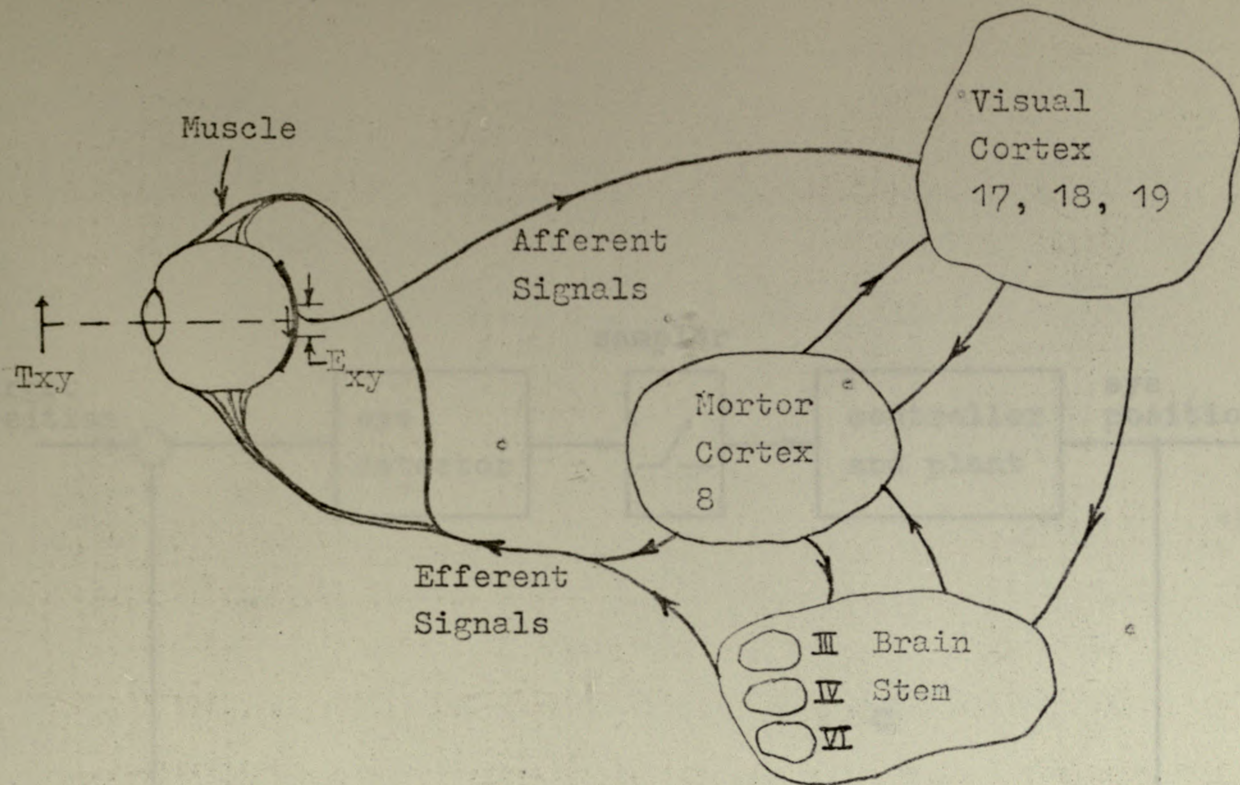
(4) Latour's difference phenomenon was reviewed here. This phenomenon implied a decreased reaction time as the experimental runs get longer.

(5) Forster's sampl-data was also reviewed. This target-asynchronized model could explain the dependence of "the percentage of abnormal response " upon the stimulus frequency. But the experiemental results possessed both target-synchronized and target-asynchronized properties.

Based on the discussion, some possible future directions were suggested. These directions included longer session experimental runs, two-dimensional tracking measurements, comparison between continuous and discontinuous stimuli, verification of Sugie's model, and relationship between eye muscle mechanism and eye tracking movement.

REFERENCE

1. D. J. Dallos, and R. W. Jones, "Learning Behavior of The Eye Fixation Control System", IEEE Trans. on Automatic control, vol. AC-8, pp. 218-227, July 1963.
2. N. Sugie, "A Model of Predictive Control in Visual Target Tracking". IEEE Trans. on System, Men, and Cybernetics, vol. SMC-1, no. 1, Jan. 1971. pp. 2-7.
3. G. J. St-Cyr and D. H. Fender, " Nonlinearities of The Human Oculomotor System Time Delay". Vision Res. vol. 9, pp. 1491-1503. Pergamon Press 1969.
4. L. Stark, G. Vossius, and L. R. Young, IEEE, Trans. on Human Factor in Electronics HFE-3, pp. 52-57. 1962.
5. L. R. Young, and L. Stark, IEEE Trans. on Human Factors in Electronics, HFE-4, pp. 38-51. 1963.
6. L. Stark, "Neurological Control System, Studies in Bioengineering". Plenum Press, New York, pp. 240-244, 351-357. 1968.
7. G. W. Lang, " Representation of The Human Operator As A Sampled- Data System", PhD Thesis, University of London, 1967.
8. S. M. Forster, S. M. Thesis Report MIT-68-2, Man Vehical Laboratory, Center for Space Research, MIT, Cambridge, Mass., 1968.
9. P. L. Latour, and M. A. Bouman, "A Non-analog Time Component in Eye Pursuit Movement". Proc. MIT Symposium, 1959.
10. P. L. Latour, "The Eye and Its Timimg". 1961.



Afferent Signals : continuous.
Efferent Signals : continuous and discret.

Fig. 1

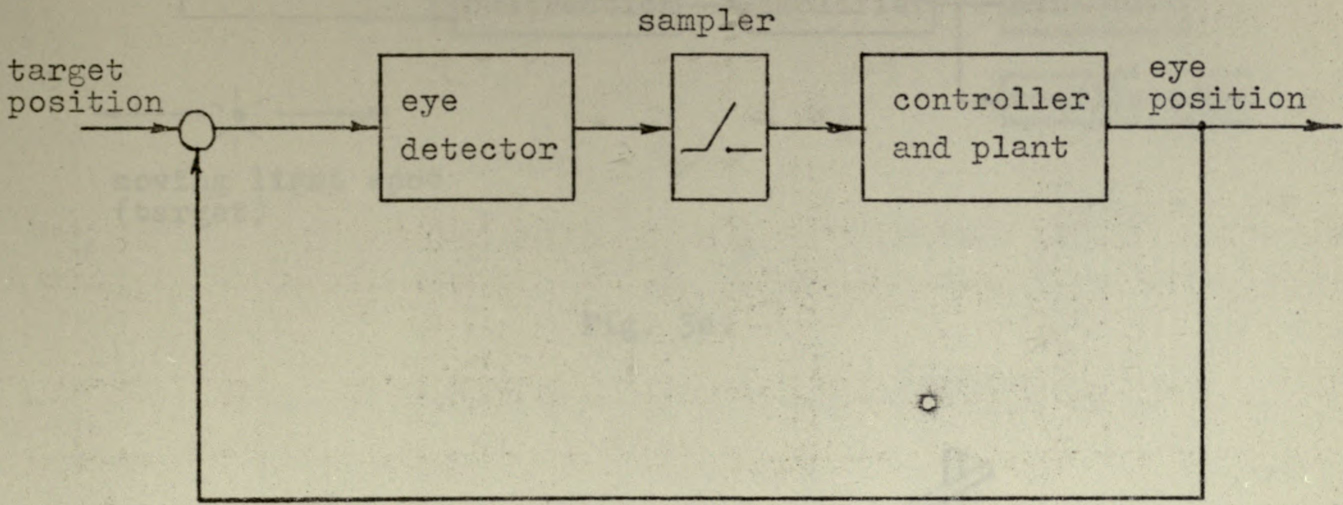

sampler
target position
eye detector
controller and plant
eye position

Fig. 2

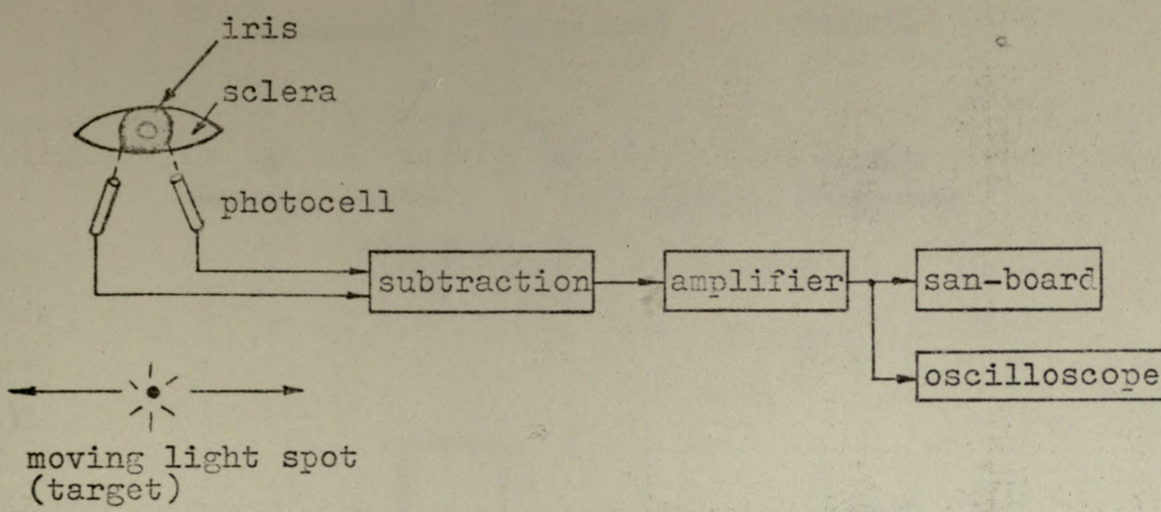

iris
sclera
photocell
subtraction
amplifier
san-board
oscilloscope
moving light spot
(target)

Fig. 3a.

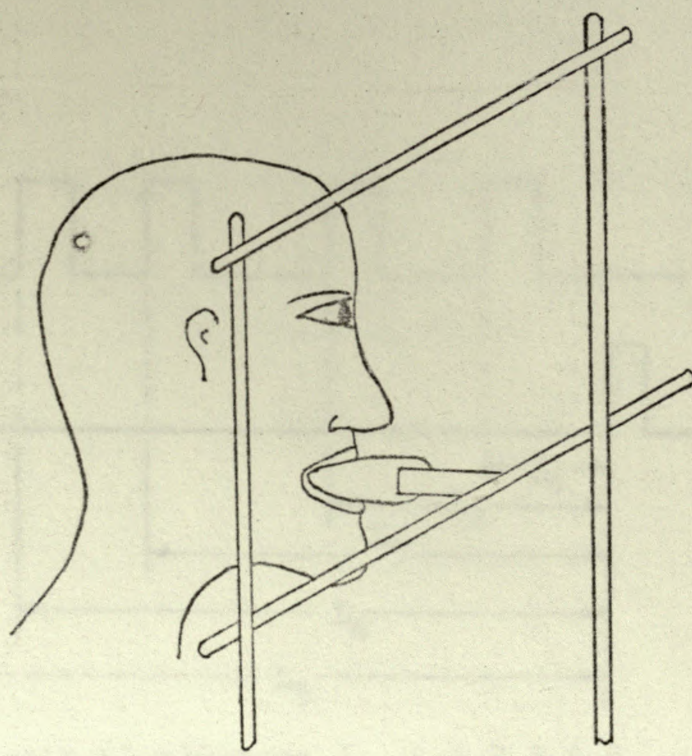
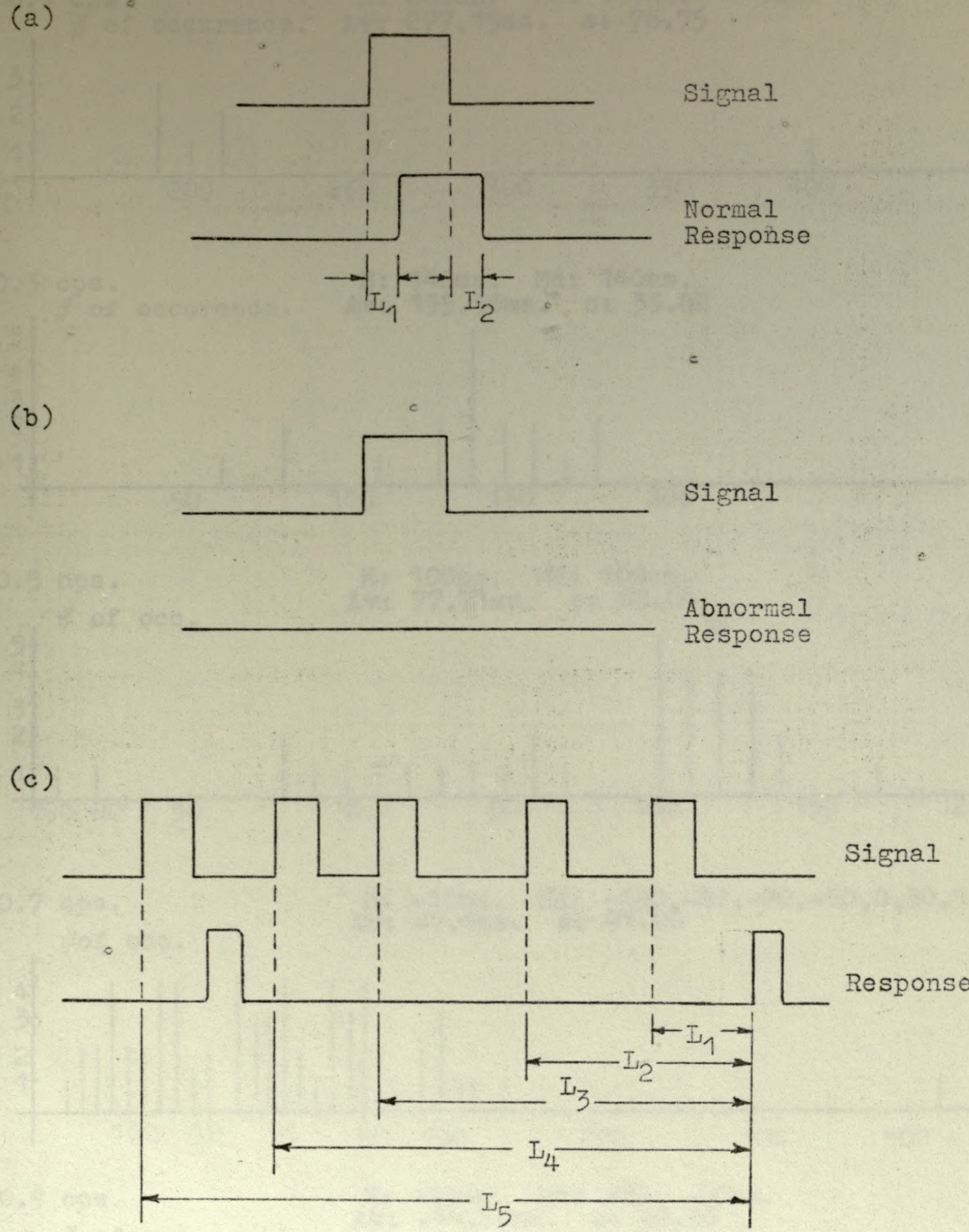


Fig. 3b.



$L=\min.(L_i>150\text{msec.}),\quad i=1,2,3,4,5,\ldots$

Fig.4

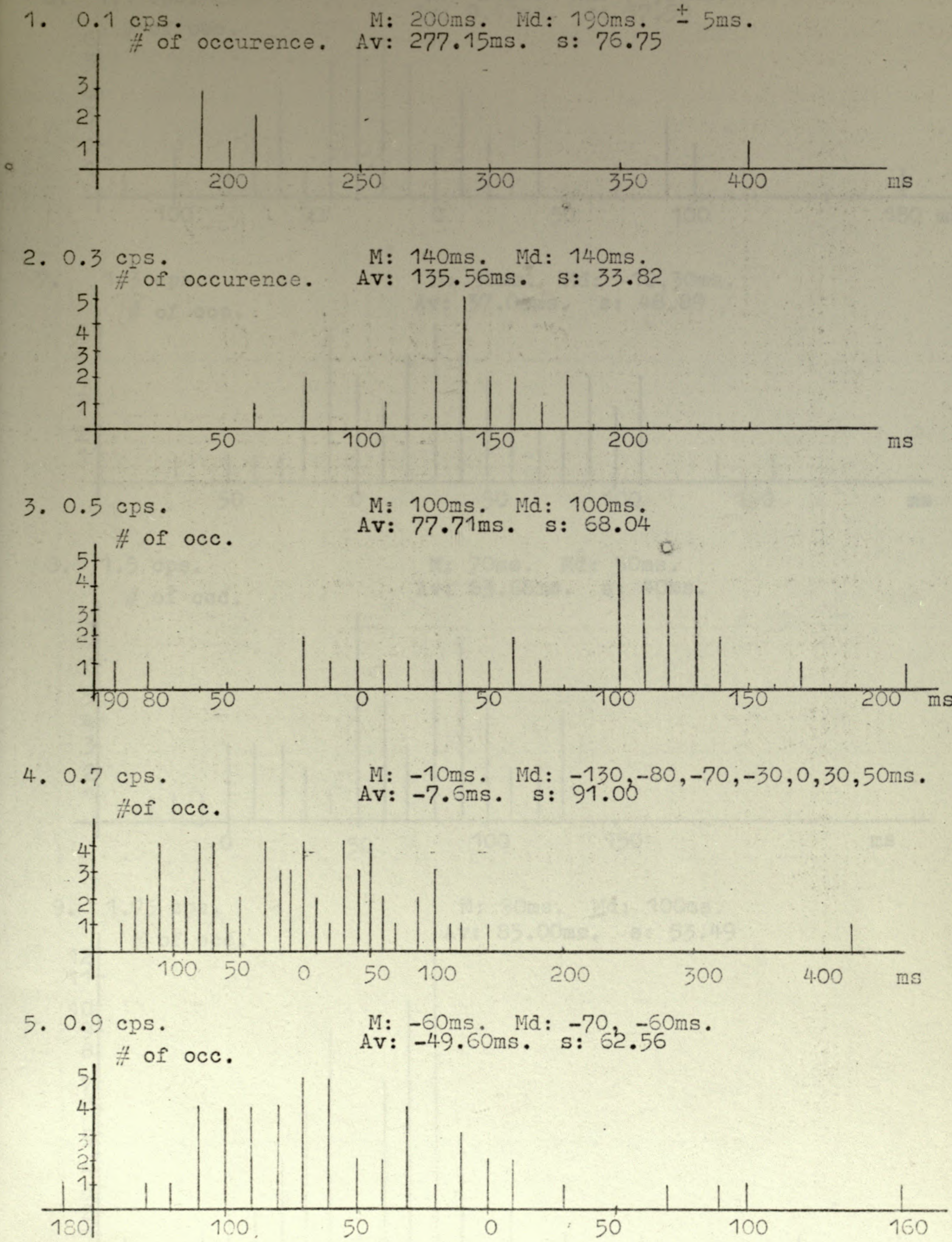
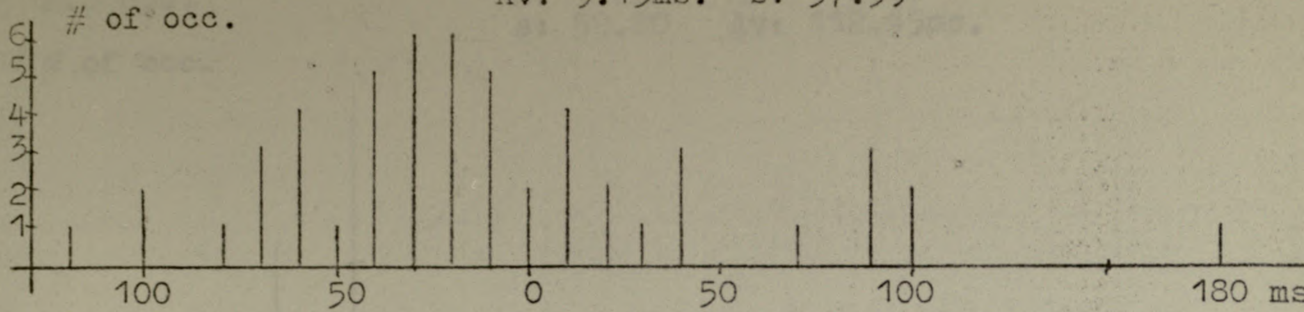


Fig.5: Predictable square wave; Subject: Mike.

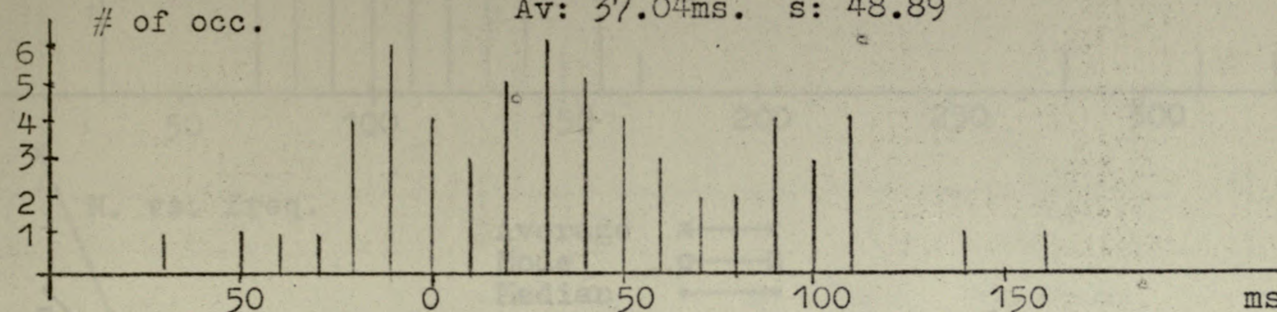
6. 1.1 cps.

M: -20ms. Md: 70,80ms.
Av: 9.43ms. s: 57.59



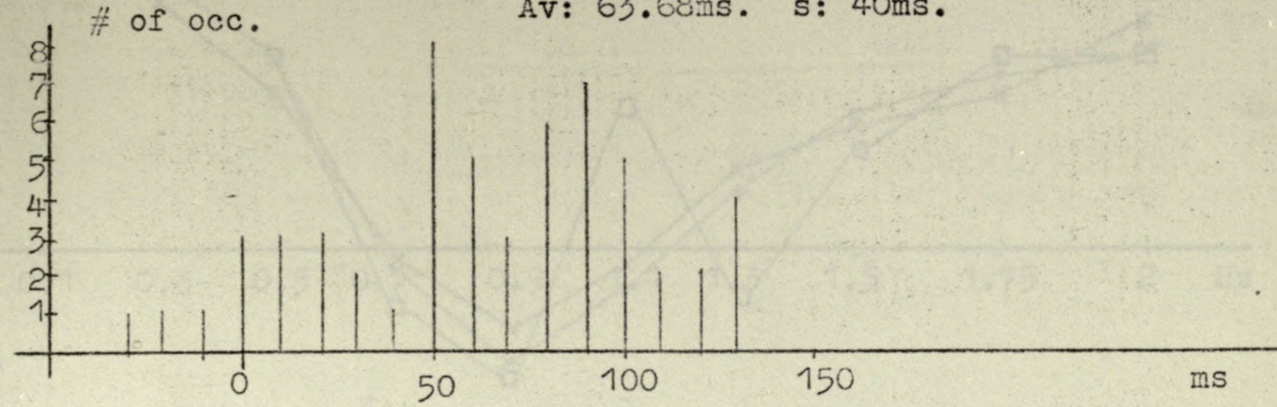
7. 1.3 cps.

M: 30ms. Md: -10,30ms.
Av: 37.04ms. s: 48.89



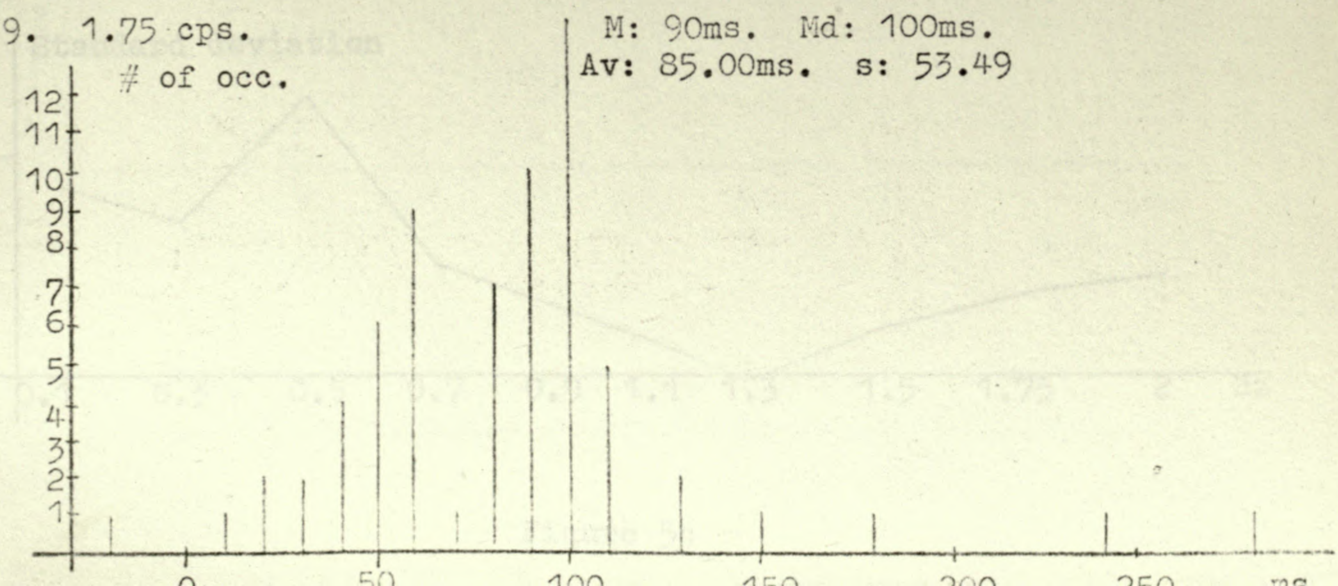
8. 1.5 cps.

M: 70ms. Md: 50ms.
Av: 63.68ms. s: 40ms.



9. 1.75 cps.

M: 90ms. Md: 100ms.
Av: 85.00ms. s: 53.49



10. 2.0 cps.

M: 100ms. Md: 100ms.
s: 59.80 Av: 112.45ms.

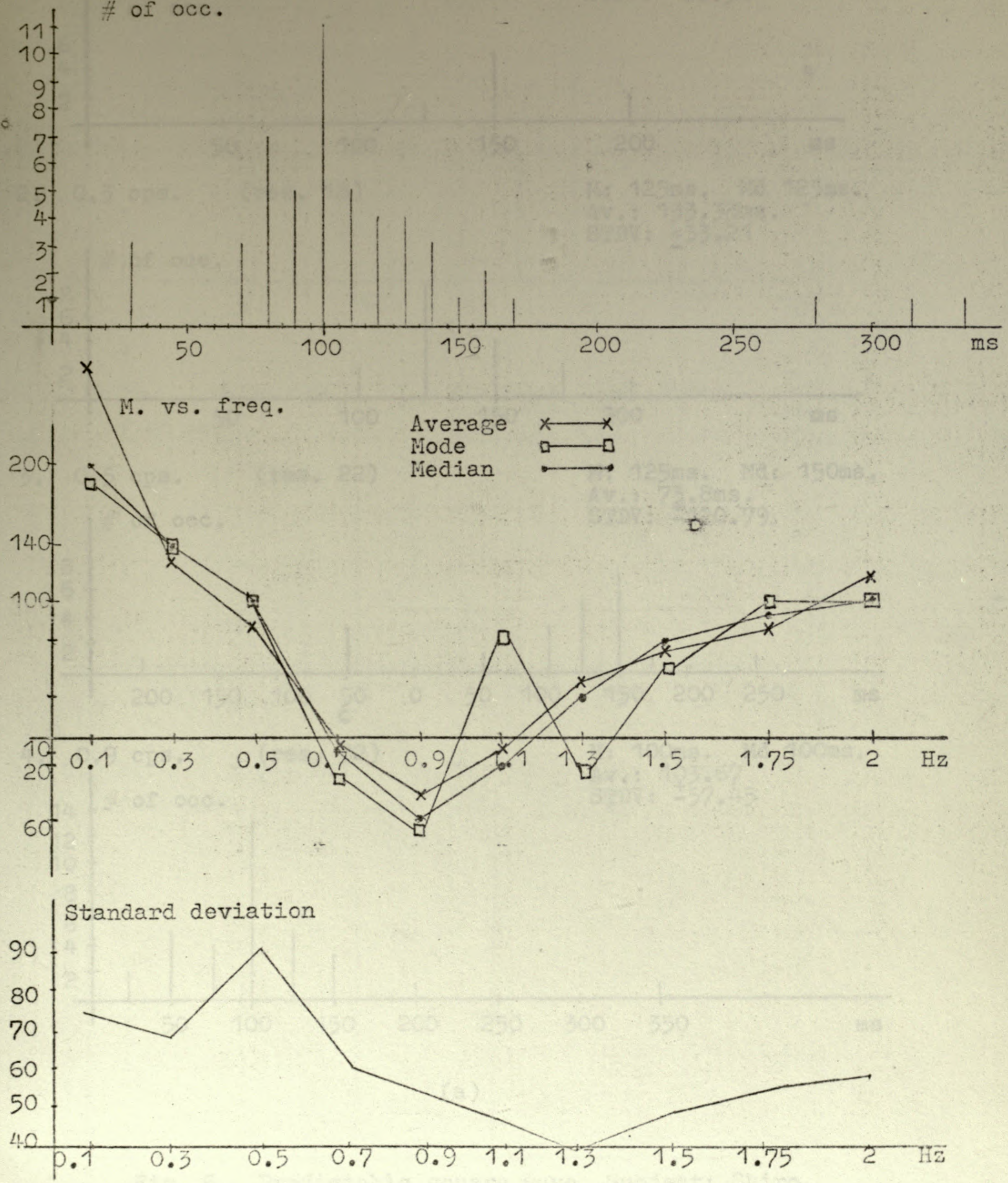
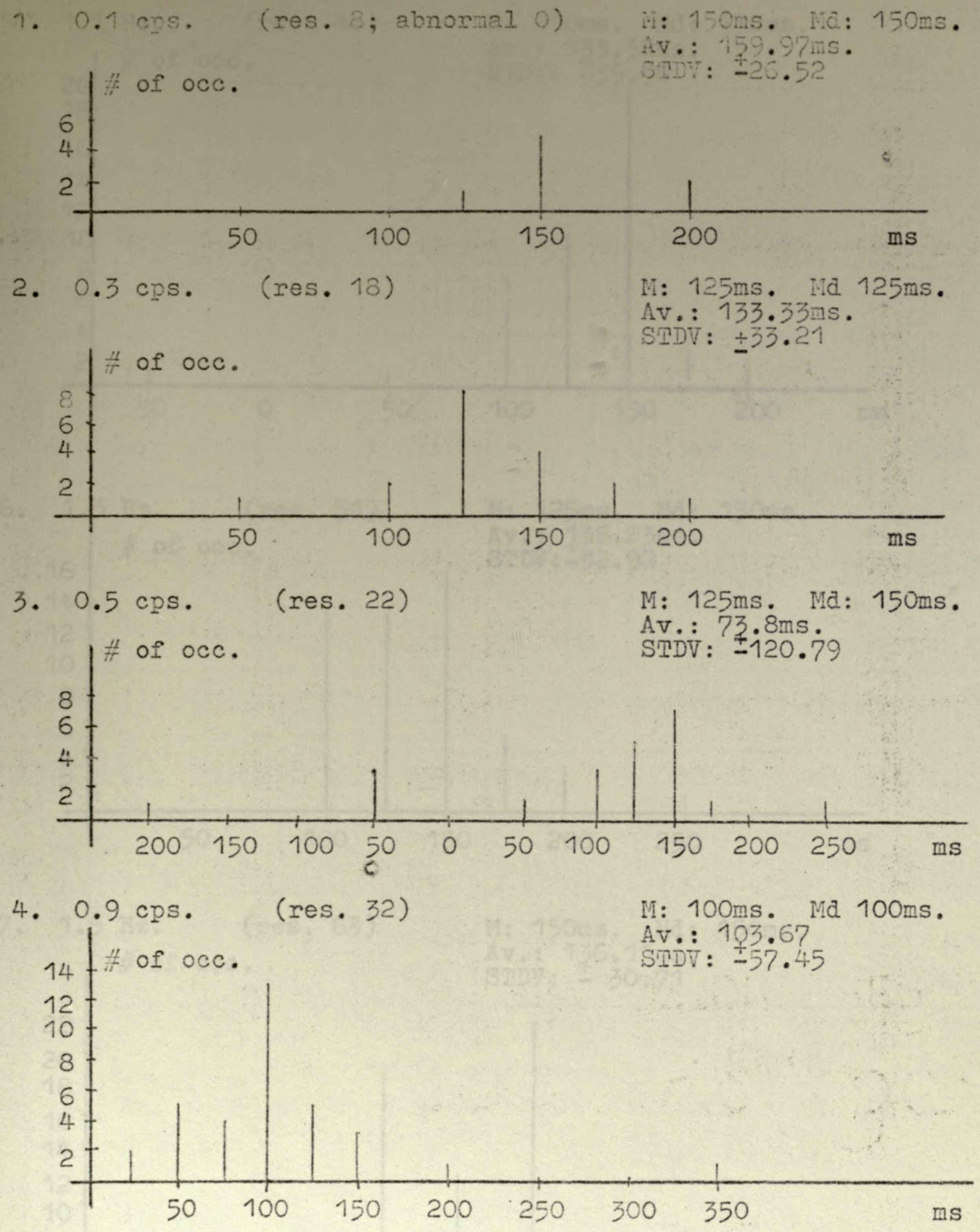


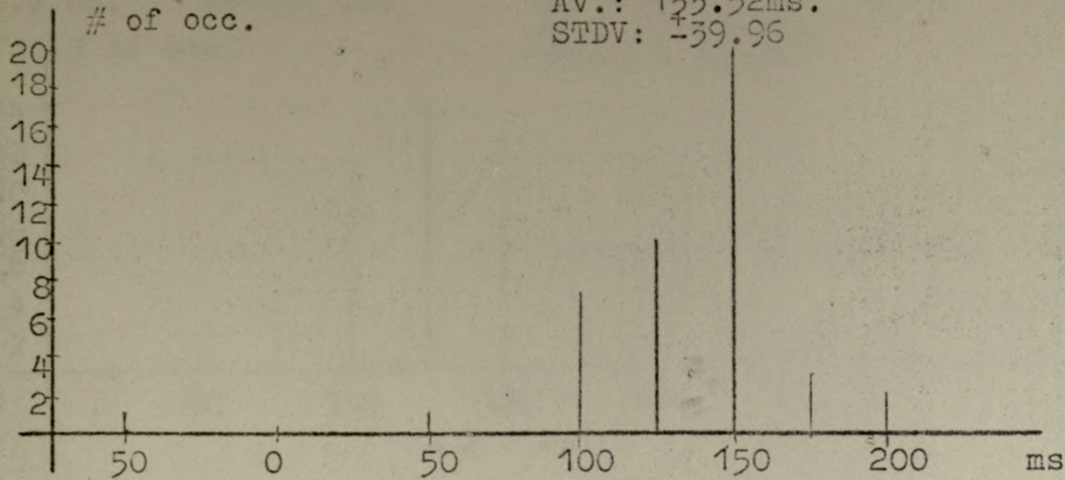
Figure 5c



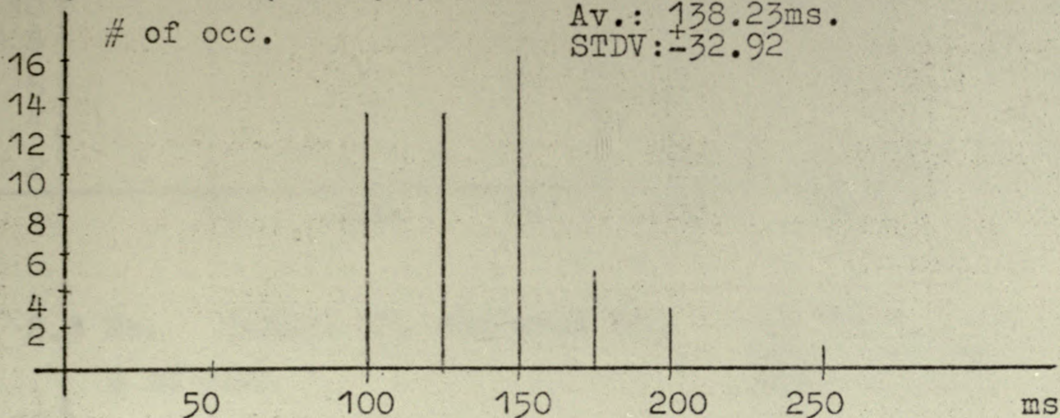
(a)

Fig. 6. Predictable square wave. Subject: Shiro.

5. 1.1 Hz. (res. 44) M: 150ms. Md: 150ms.
Av.: 133.52ms.
STDV: ±39.96



6. 1.3 Hz. (res. 51) M: 125ms. Md: 150ms.
Av.: 138.23ms.
STDV: ±32.92



7. 1.5 Hz. (res. 63) M: 150ms. Md: 150ms.
Av.: 136.11ms.
STDV: ± 30.71

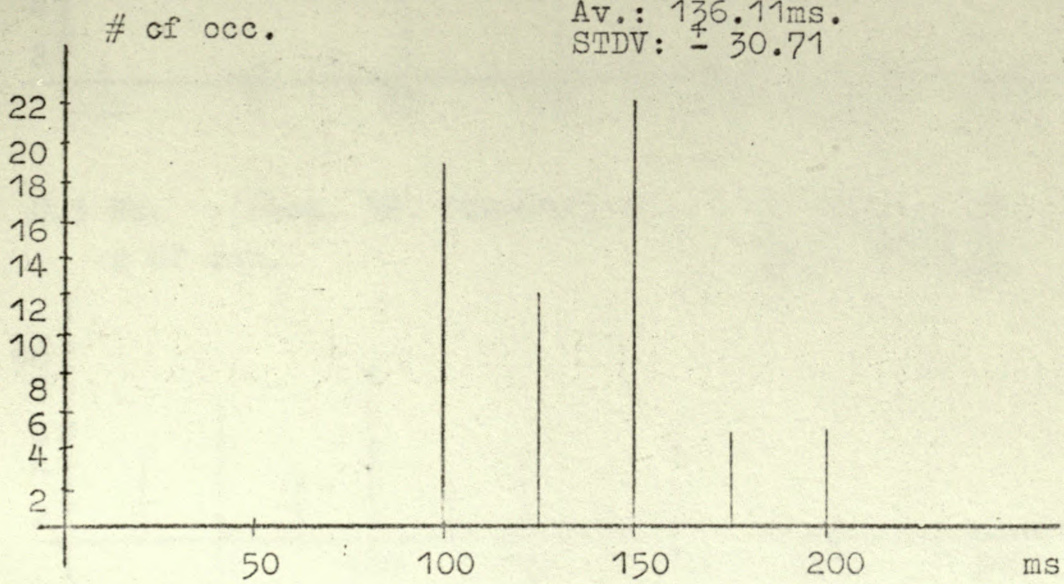
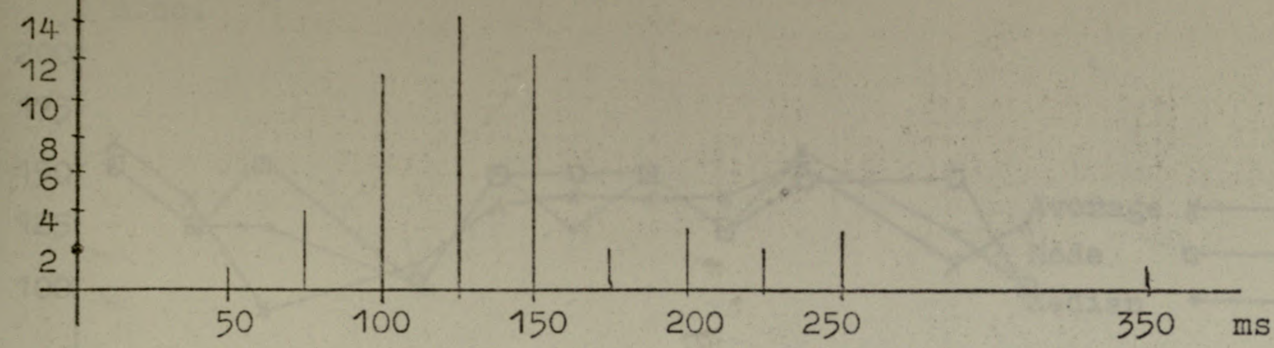


Fig. 6b.

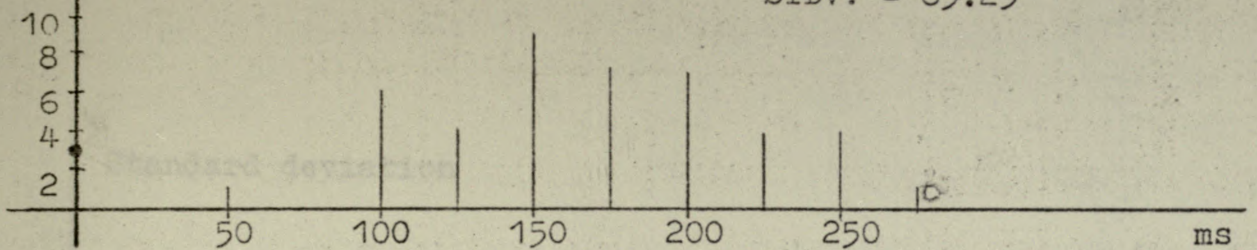
8. 1.7 Hz. (resp. 55) M: 125ms. Md: 125ms.
Av.: 136.36ms.
STDV: ± 59.86

# of occ.



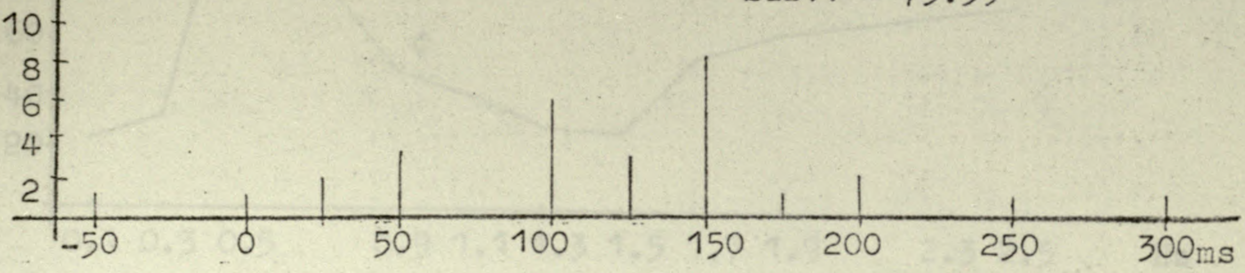
9. 1.9 Hz. (resp. 46, abnormal 33) M: 162.5ms. Md: 150ms.
Av.: 158.69ms.
STDV: ± 65.23

# of occ.



10. 2.3 Hz. (resp. 29, abnormal 48) M: 125ms. Md: 150ms.
Av.: 118.96ms.
STDV: ± 73.39

# of occ.



11. 2.5 Hz. (resp. 52, abnormal 80) M: 100ms. Md: 100ms.
Av.: 126.08ms.
STDV: ± 77.80

# of occ.

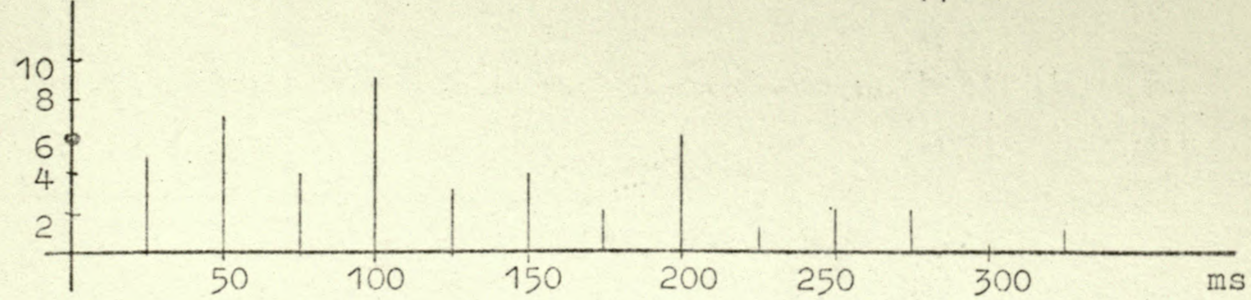
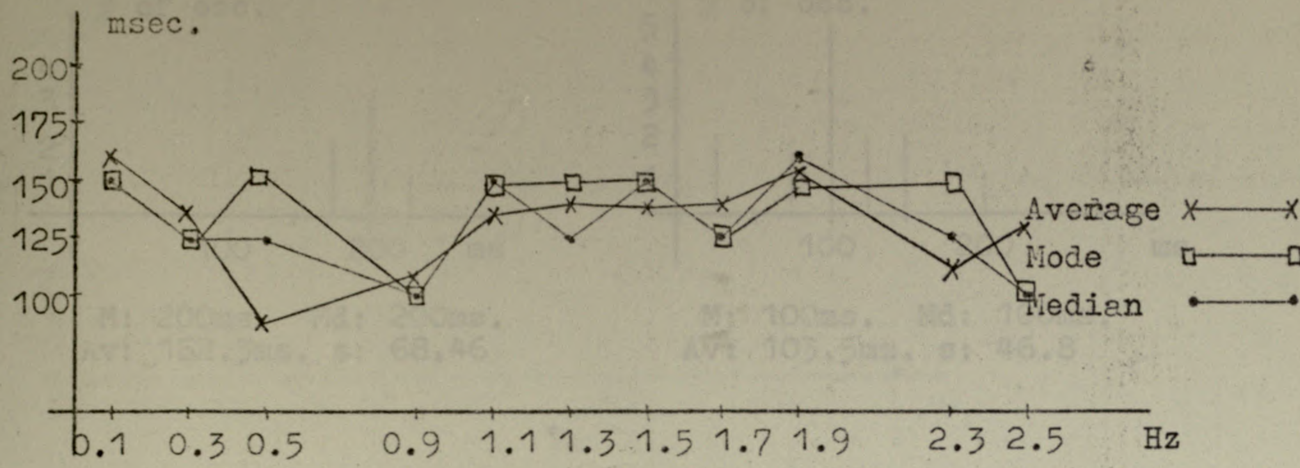


Fig. 6c.


msec.
200
175
150
125
100
Average
Mode
Median
0.1 0.3 0.5 0.9 1.1 1.3 1.5 1.7 1.9 2.3 2.5 Hz

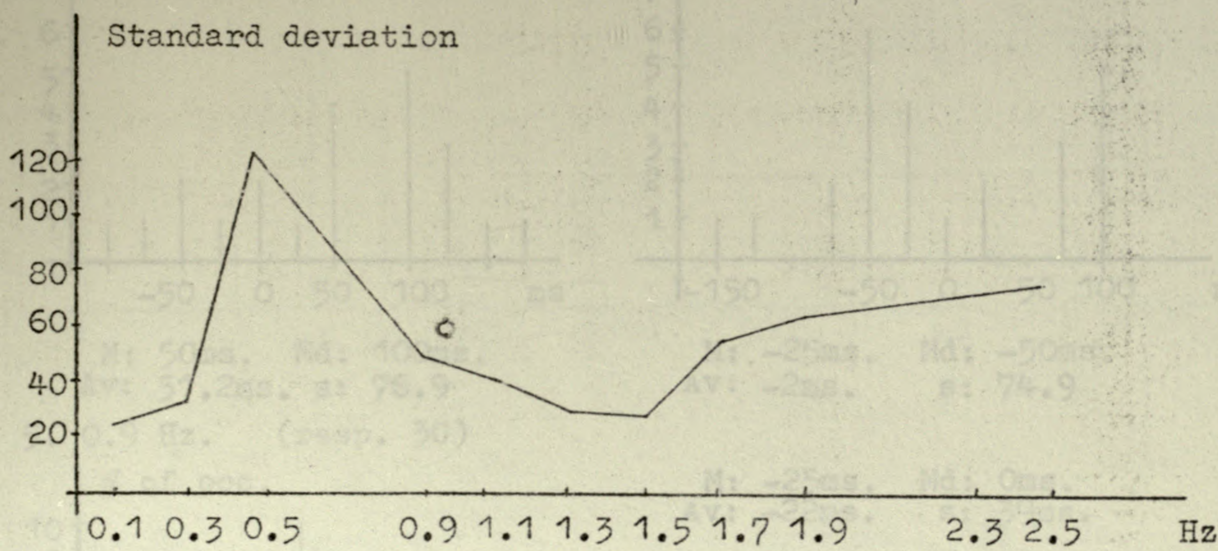
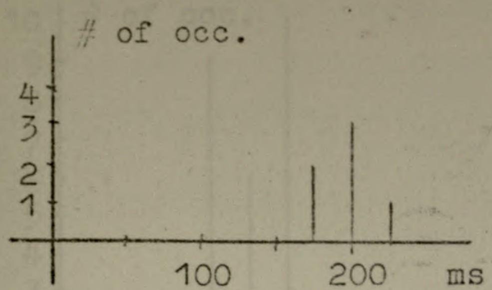

Standard deviation
120
100
80
60
40
20
0.1 0.3 0.5 0.9 1.1 1.3 1.5 1.7 1.9 2.3 2.5 Hz

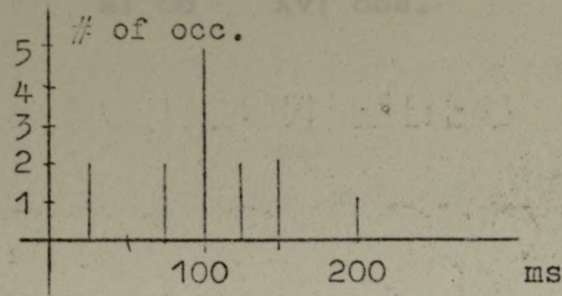
Fig. 6d.

1. 0.1 Hz. (resp. 6)



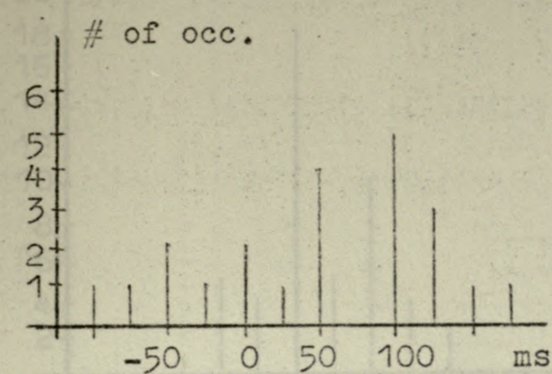
M: 200ms. Md: 200ms.
Av: 162.5ms. s: 68.46

2. 0.3 Hz. (resp. 16)



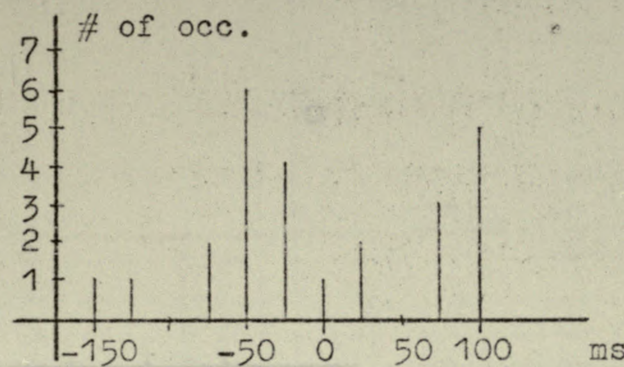
M: 100ms. Md: 100ms.
Av: 103.5ms. s: 46.8

3. 0.5 Hz. (resp. 22)



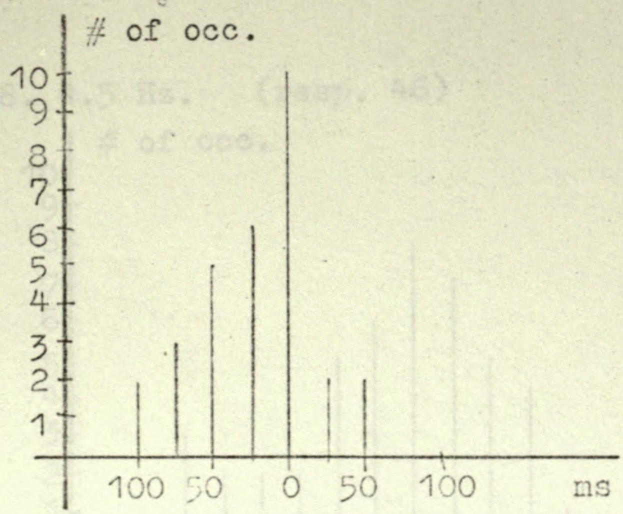
M: 50ms. Md: 100ms.
Av: 51.2ms. s: 76.9

4. 0.7 Hz. (resp. 25)



M: -25ms. Md: -50ms.
Av: -2ms. s: 74.9

5. 0.9 Hz. (resp. 30)



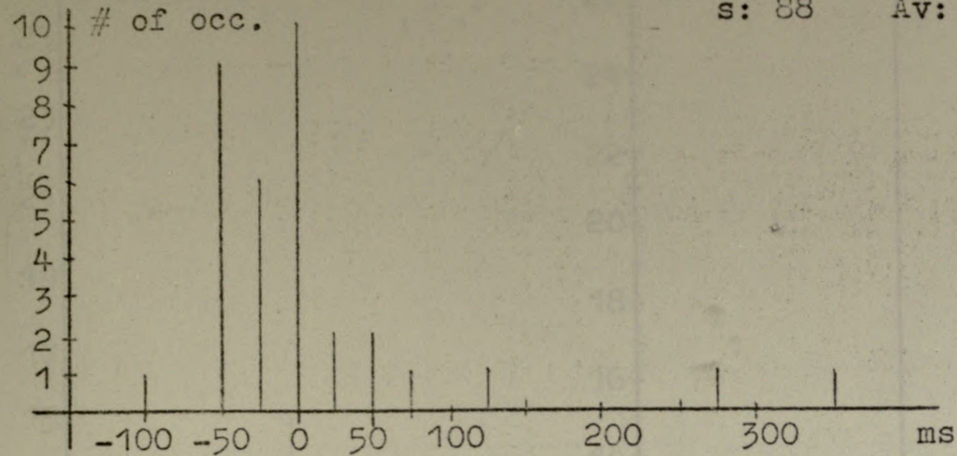
M: -25ms. Md: 0ms.
Av: -22ms. s: 39ms.

(a)

Fig. 7: Predictable square wave. Subject: S.

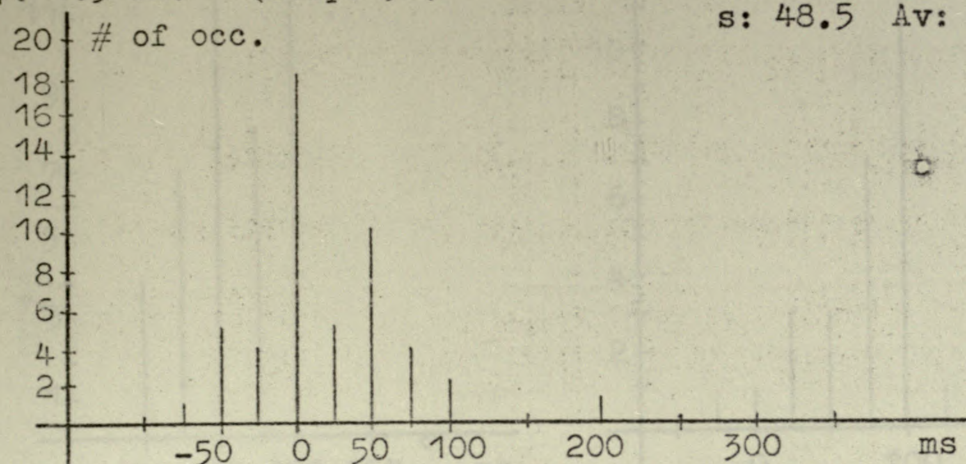
6. 1.1 Hz. (resp. 34)

M: Oms. Md: Oms.
s: 88 Av: 8ms.



7. 1.3 Hz. (resp. 50)

M: Oms. Md: Oms.
s: 48.5 Av: 18ms.



8. 1.5 Hz. (resp. 46)

M: 25ms. Md: 25ms.
s: 68.8 Av: 14.6ms.

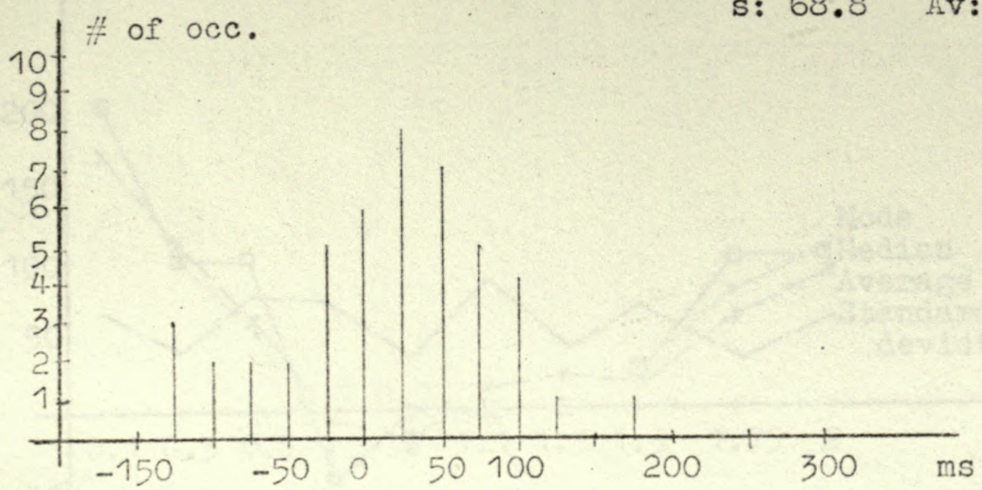
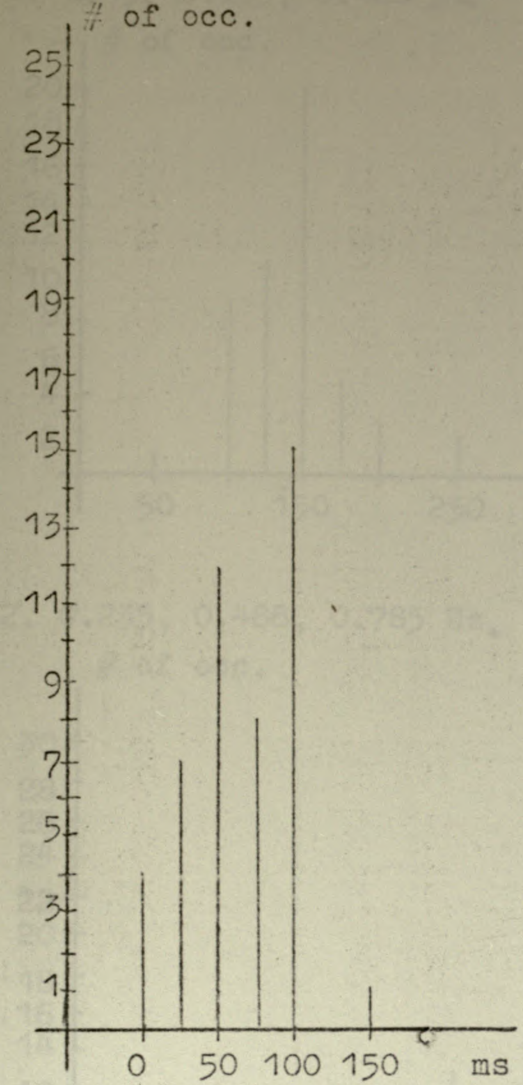


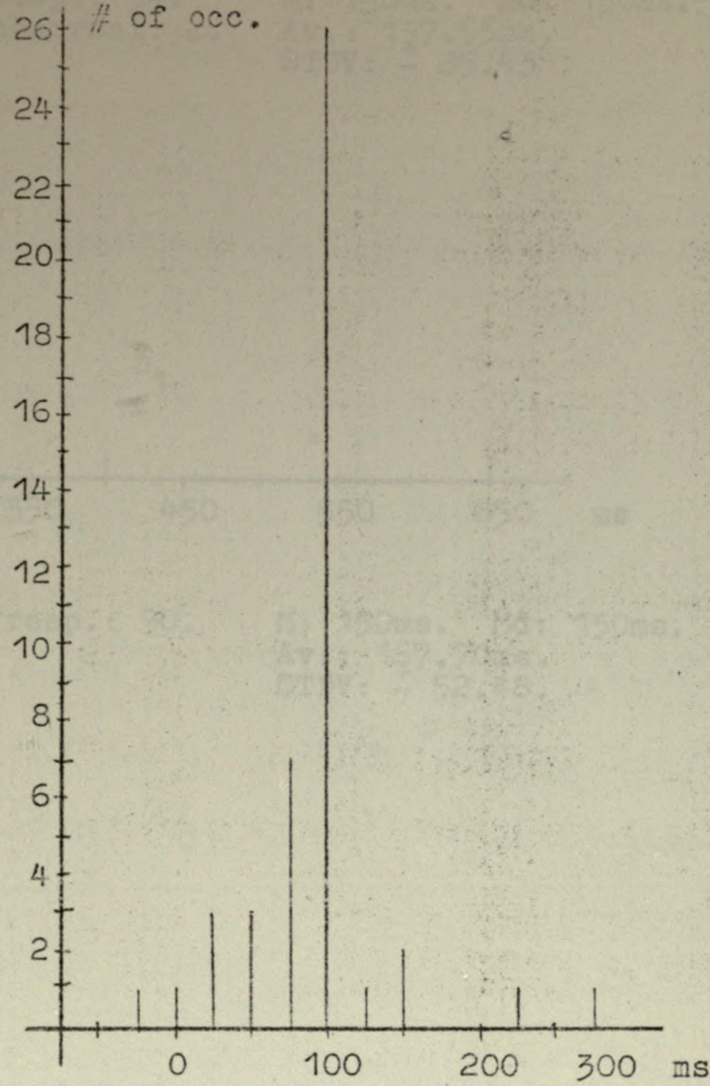
Fig. 7b.

9. 1.75 Hz. (resp. 47)



M: 75ms. Md: 100ms.

10. 2 Hz. (resp. 46)



M: 100ms. Md: 100ms.

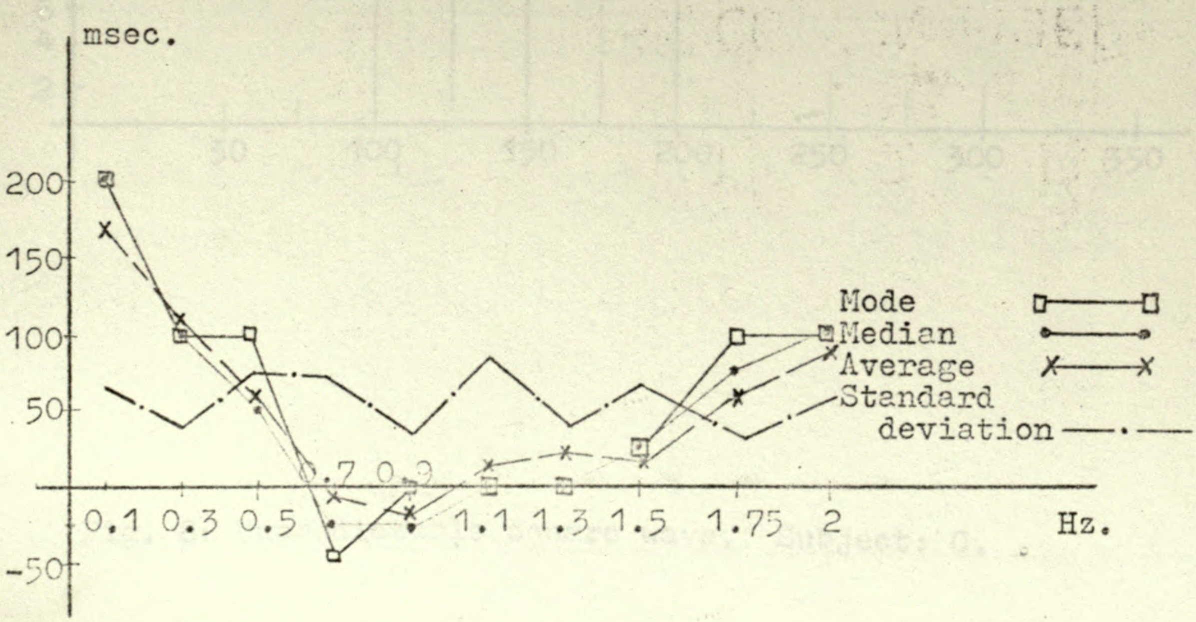
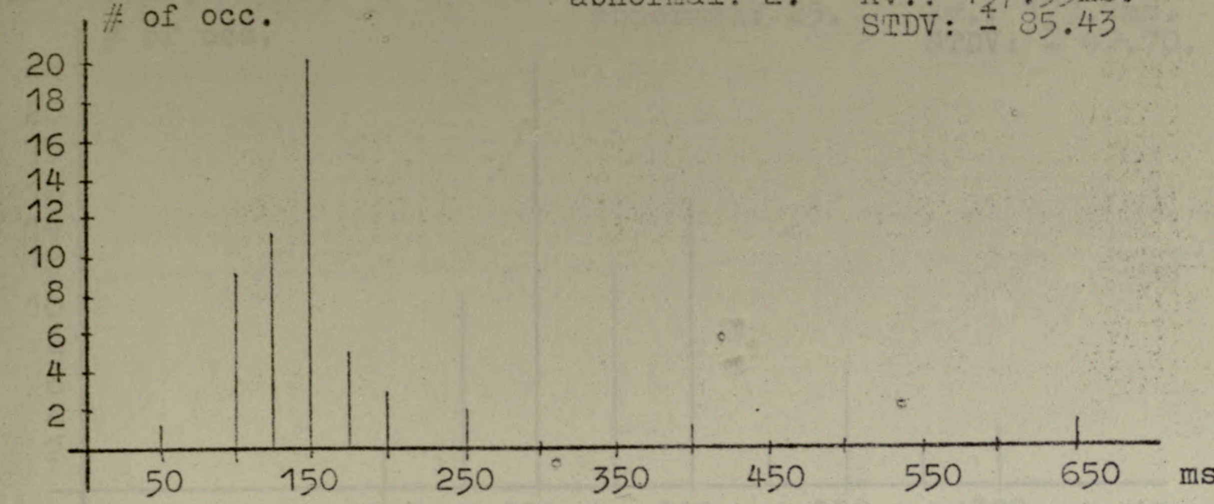


Fig. 7c.

1. 0.13, 0.235, 0.488 Hz. resp.: 54. M: 150ms. Md: 150ms.
abnormal: 2. Av.: 157.55ms.
STDV: ± 85.43



2. 0.235, 0.488, 0.785 Hz. resp.: 90. M: 150ms. Md: 150ms.
Av.: 167.70ms.
STDV: ± 52.48.

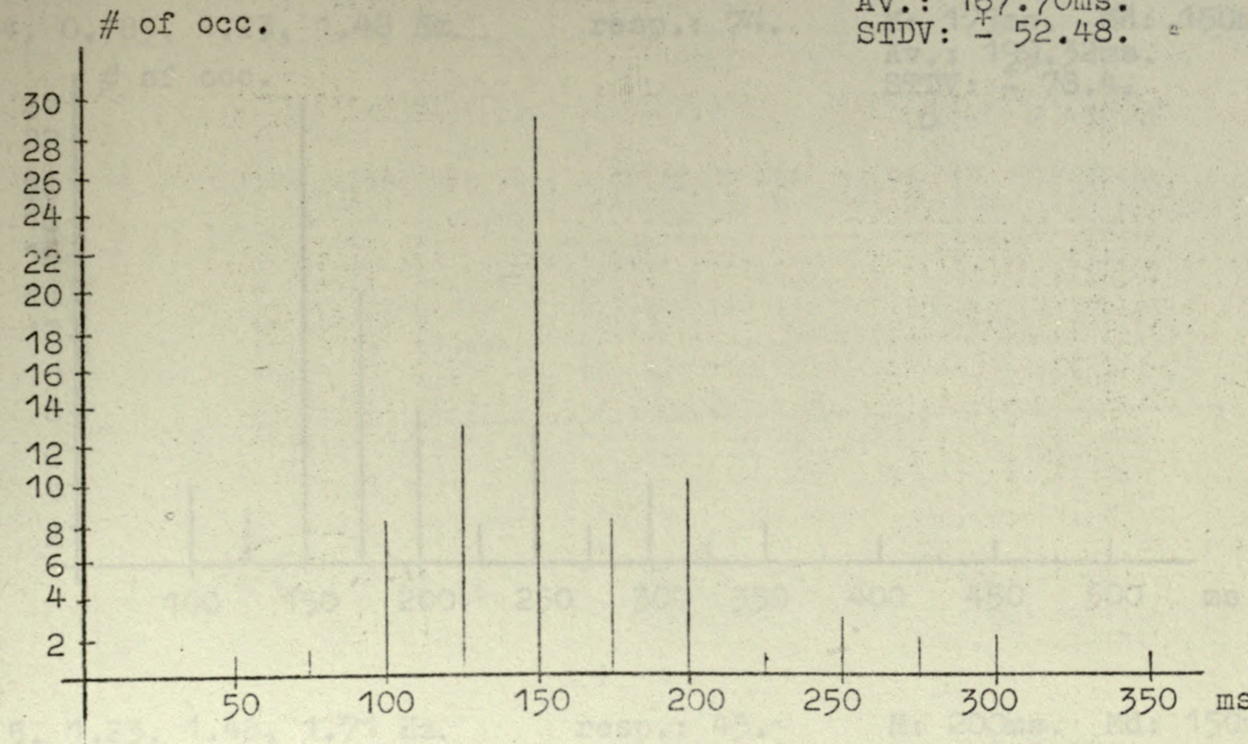
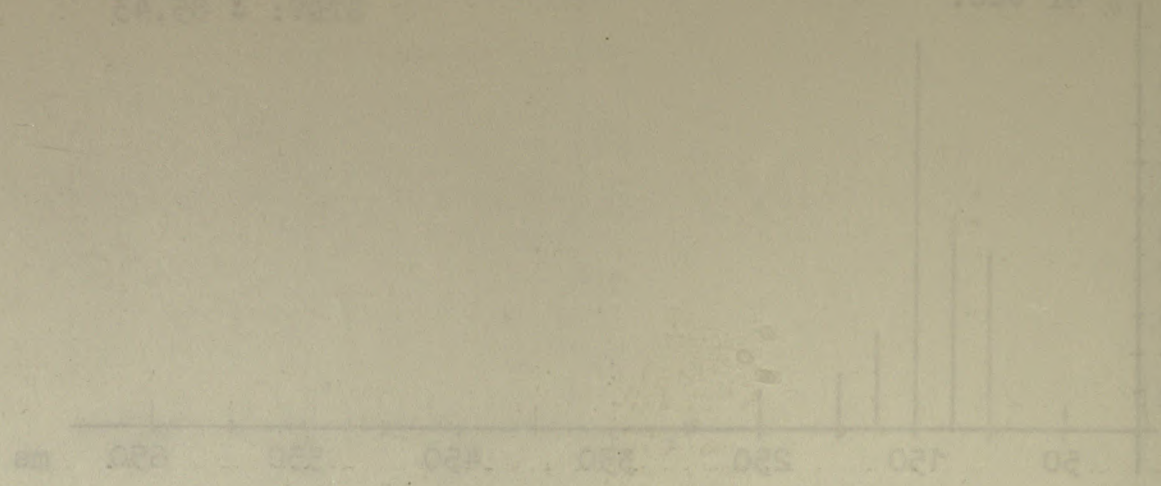


Fig. 8: Unpredictable square wave. Subject: C.

1. 0.13, 0.235, 0.488 Hz. resp.: 34. M: 150ms, Md: 150ms.
acumulation?: 2. Av.: 157.35ms.
SDEV: ± 36.45



2. 0.235, 0.488, 0.785 Hz. resp.: 90. M: 150ms. Md: 150ms.
Av.: 157.75ms.
SDEV: ± 32.48.

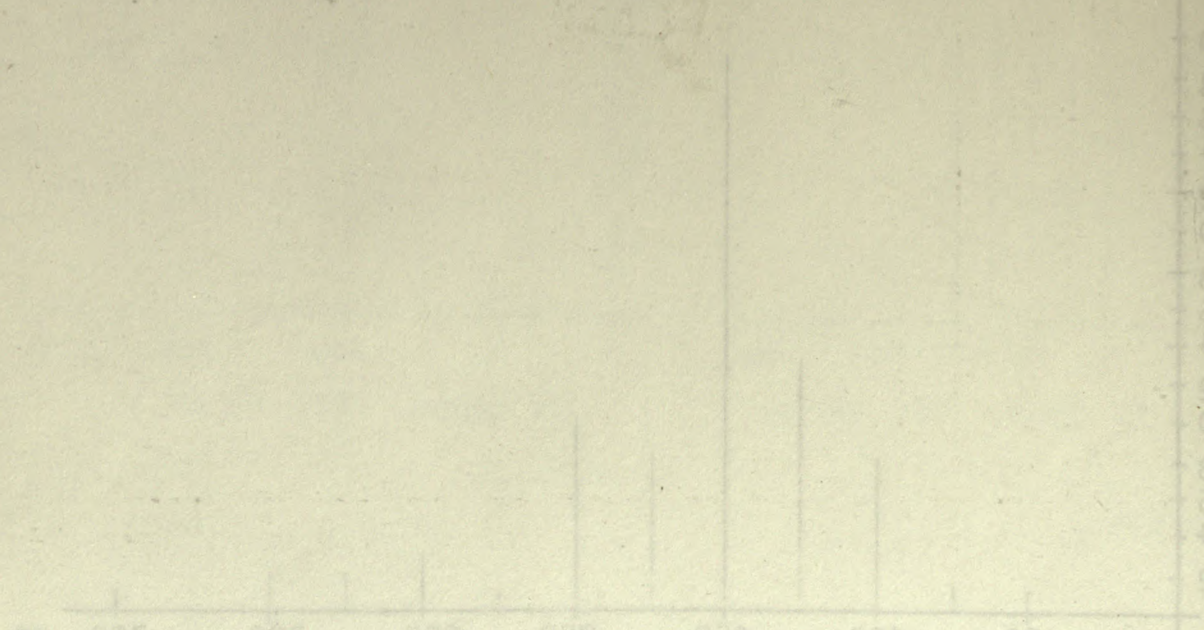
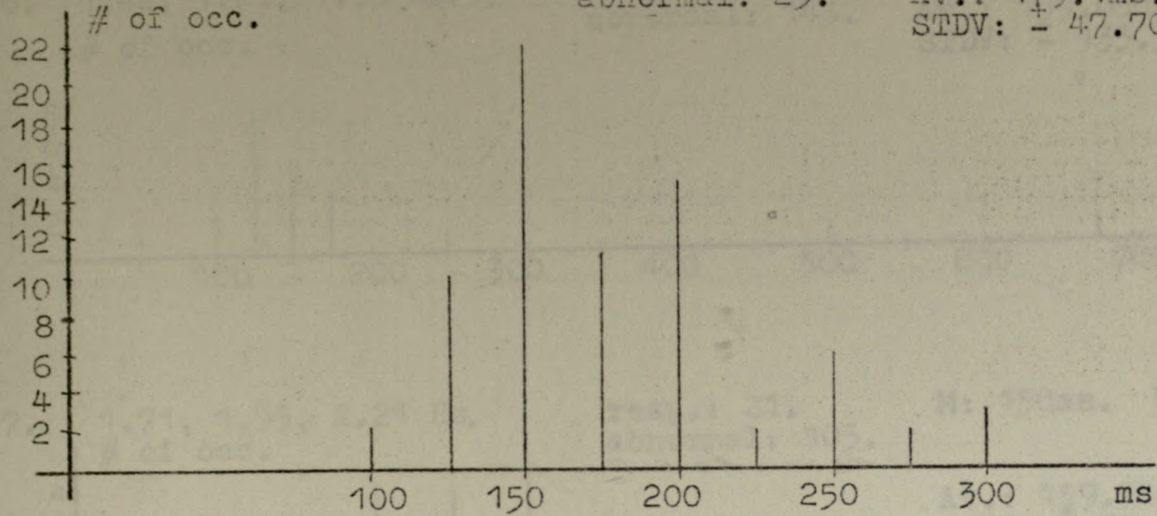
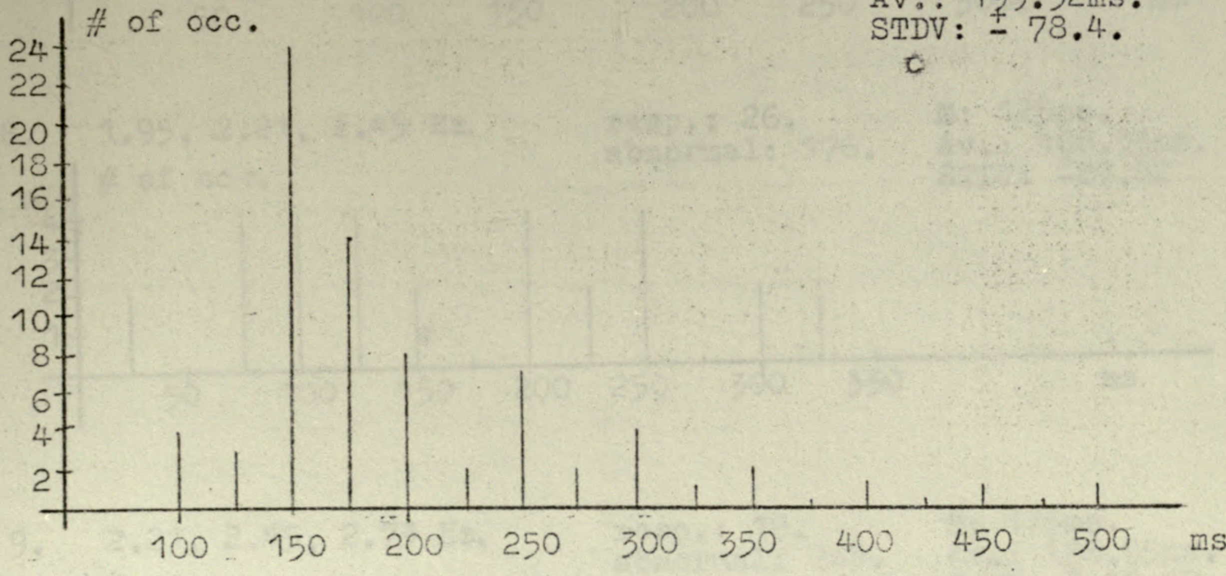


Fig. 8: Unpredictable source wave. Subject: O.

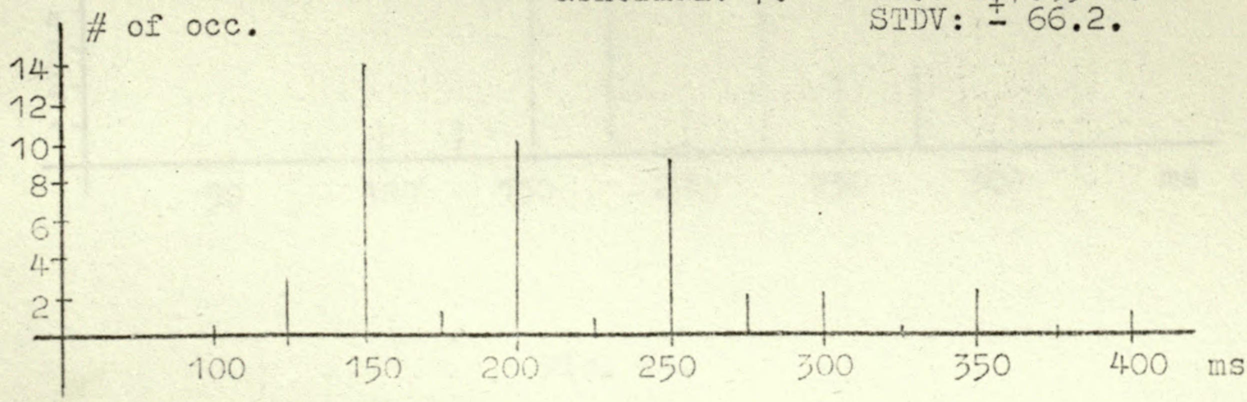
3. 0.488, 0.785, 1.23 Hz. resp.:73. abnormal: 25. M: 175ms. Md: 150ms. Av.: 179.1ms. STDV: ± 47.70.



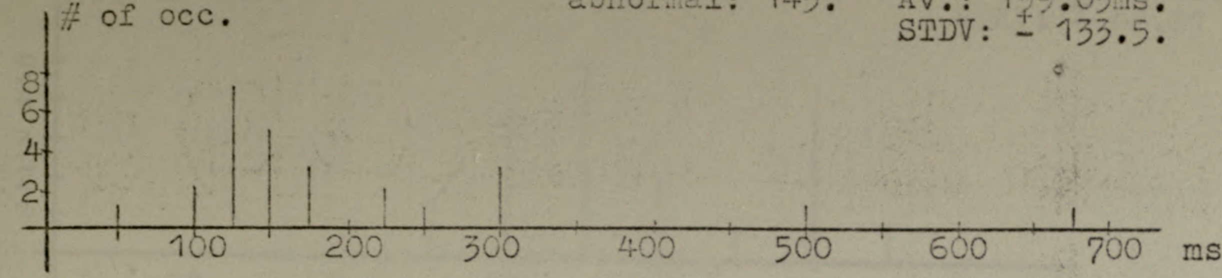
4. 0.785, 1.23, 1.48 Hz. resp.: 74. M: 175ms. Md: 150ms. Av.: 199.32ms. STDV: ± 78.4.



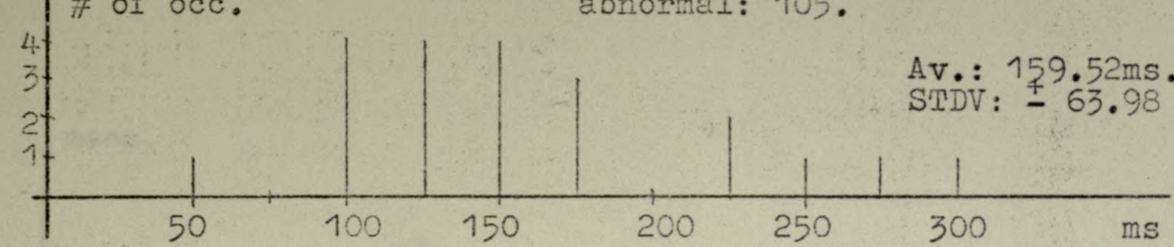
5. 1.23, 1.48, 1.71 Hz. resp.: 45. abnormal: 7. M: 200ms. Md: 150ms. Av.: 207.95ms. STDV: ± 66.2.



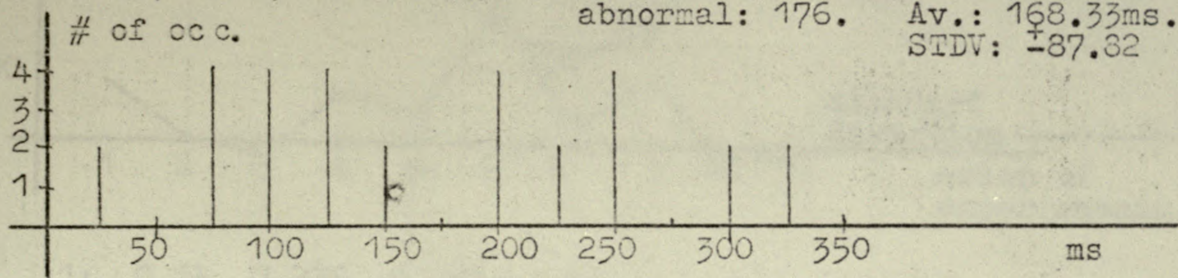
6. 1.48, 1.71, 1.95 Hz. resp.: 27. abnormal: 143. M: 150ms. Md: 125ms. Av.: 199.03ms. STDV: ± 133.5.



7. 1.71, 1.95, 2.21 Hz. resp.: 21. abnormal: 105. M: 150ms. Md: 100, 125, 150ms. Av.: 159.52ms. STDV: ± 63.98



8. 1.95, 2.21, 2.45 Hz. resp.: 26. abnormal: 176. M: 125ms. Av.: 168.33ms. STDV: ±87.82



9. 2.21, 2.45, 2.73 Hz. resp.: 19. abnormal: 285. M: 175ms. Av.: 186.95ms. STDV: ± 48.18.

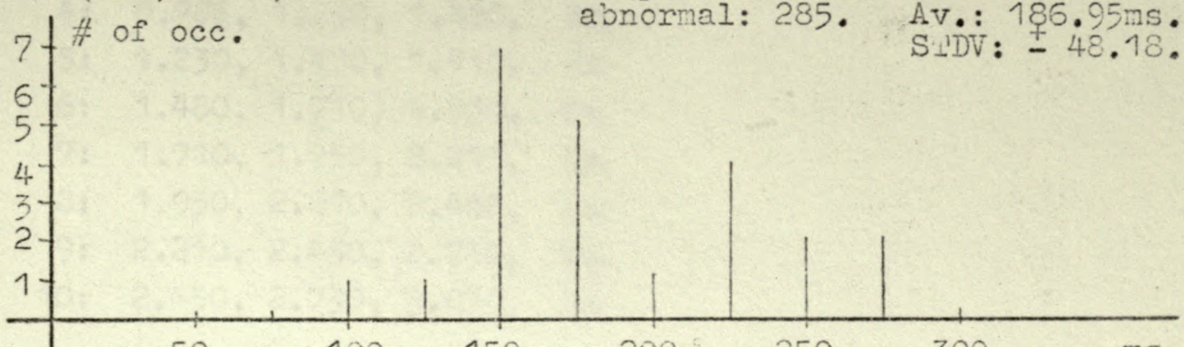
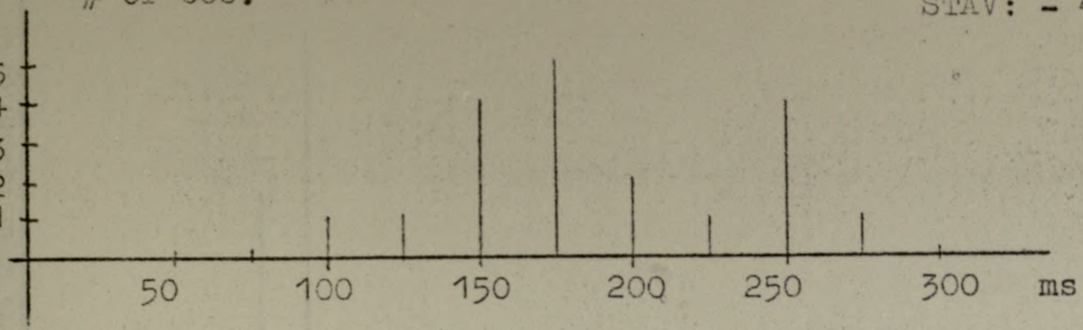
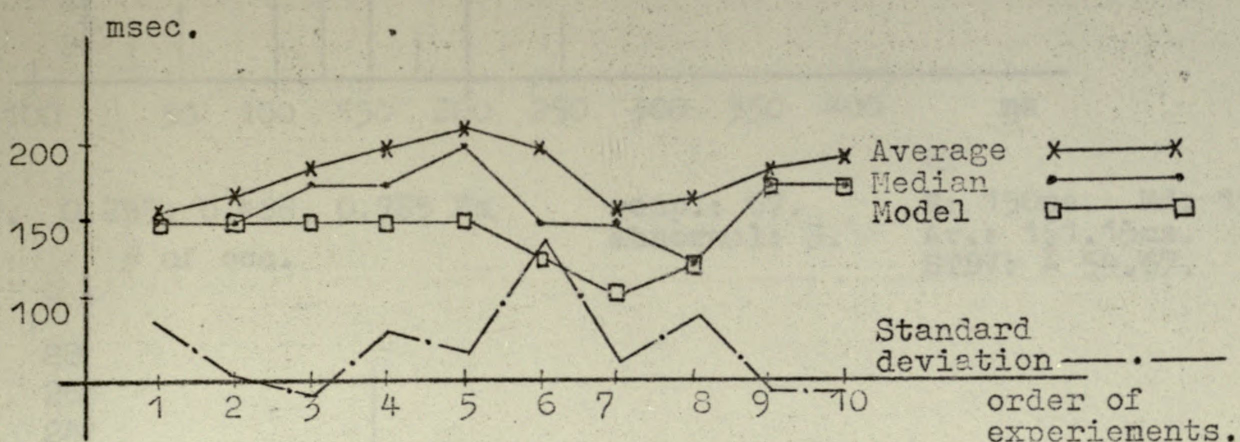


Fig. 8c.

10. 2.45, 2.73, 3.01 Hz   resp.: 19.   M: 175ms. Md: 175m
abnormal: 265.   Av.: 189.47ms.
STAV: ± 48.81

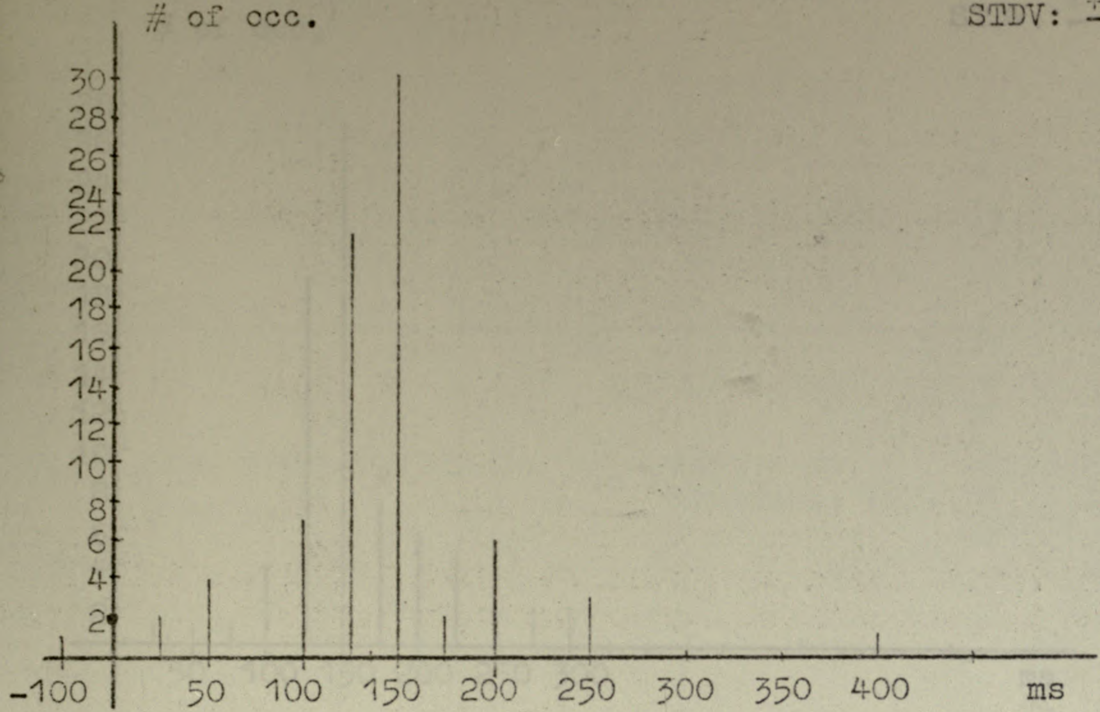




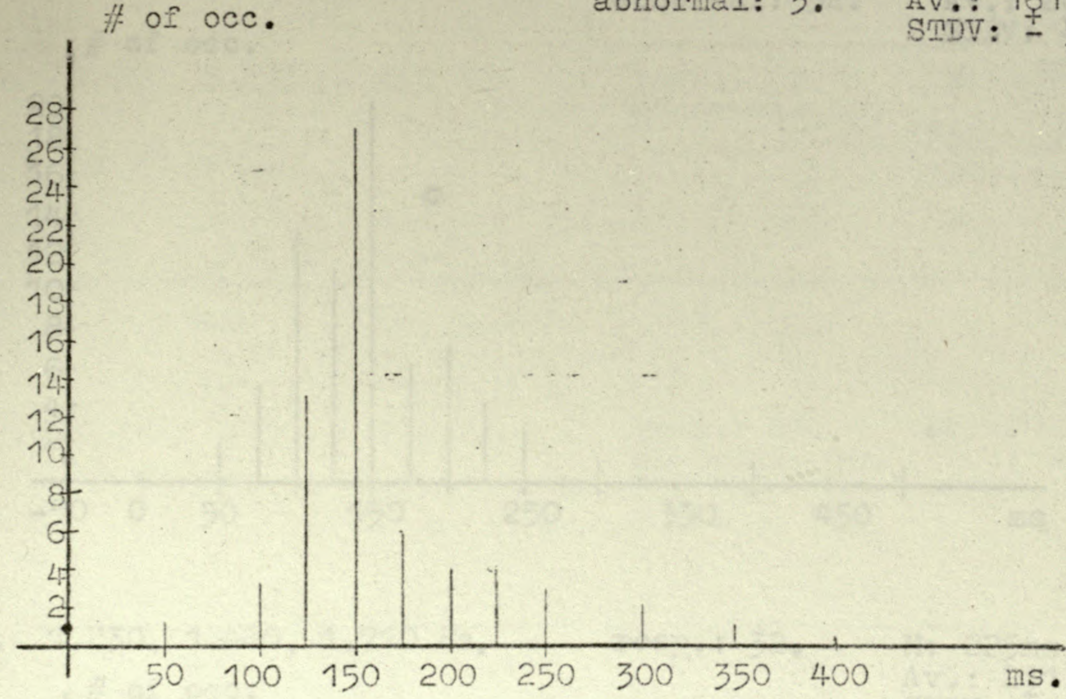
1: 0.13, 0.235, 0.488, Hz.
2: 0.235, 0.488, 0.785, Hz.
3: 0.488, 0.785, 1.230, Hz.
4: 0.785, 1.230, 1.480, Hz.
5: 1.230, 1.480, 1.710, Hz.
6: 1.480, 1.710, 1.950, Hz.
7: 1.710, 1.950, 2.210, Hz.
8: 1.950, 2.210, 2.450, Hz.
9: 2.210, 2.450, 2.730, Hz.
10: 2.450, 2.730, 3.010, Hz.

Fig. 8d.

1. 0.13, 0.235, 0.488 Hz. resp.: 80. abnormal: 10. M: 150ms. Md: 150ms. Av.: 135ms. STDV: ± 61.7



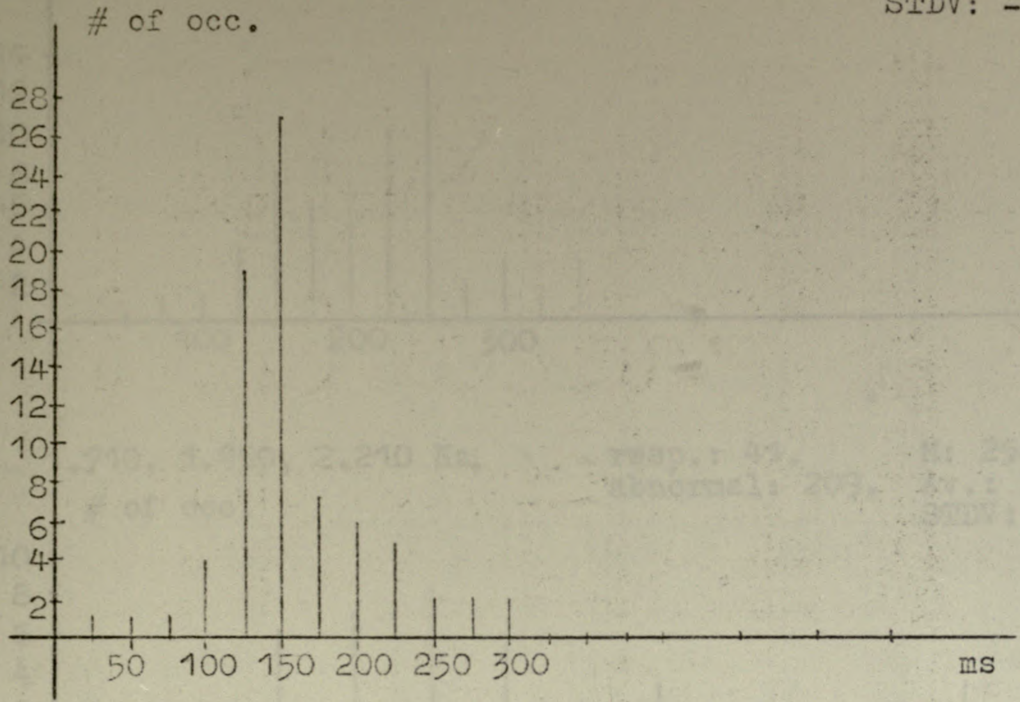
2. 0.235, 0.488, 0.785 Hz. resp.: 67. abnormal: 3. M: 150ms. Md: 150ms. Av.: 161.15ms. STDV: ± 54.67.



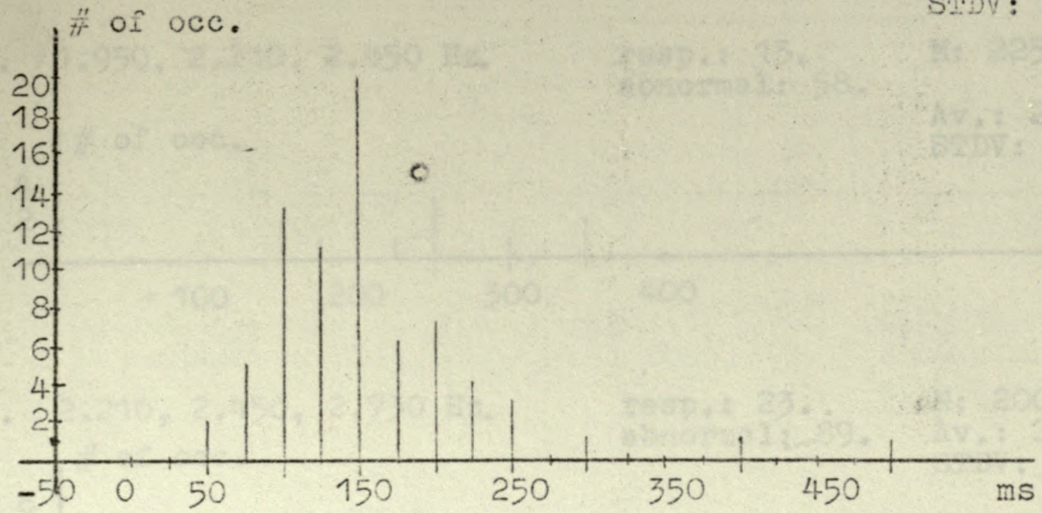
Unpredictable square wave. Subject: Shiro.

Fig. 9.

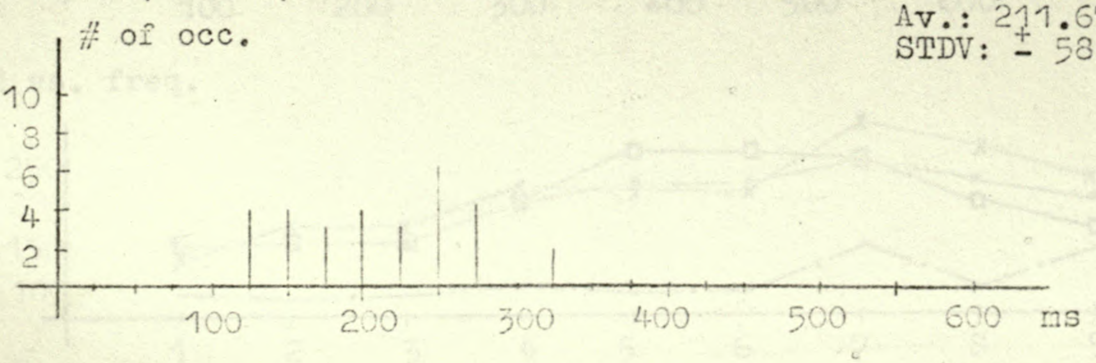
3. 0.488, 0.785, 1.230 Hz. resp.: 78. abnormal: 9. M: 150ms. Md: 150ms. Av.: 158.2ms. STDV: ± 50.24



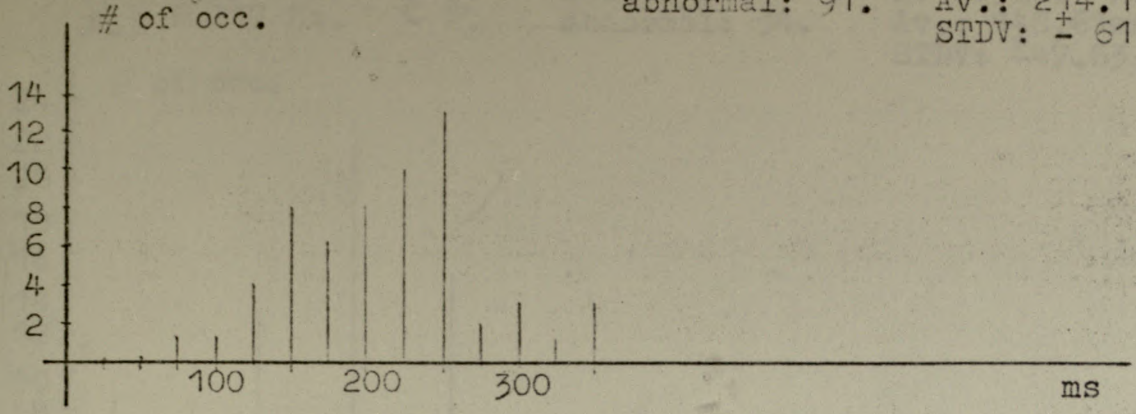
4. 0.785, 1.230, 1.480 Hz. resp.: 77. abnormal: 32. M: 200ms. Md: 200ms. Av.: 202ms. STDV: ± 74.29



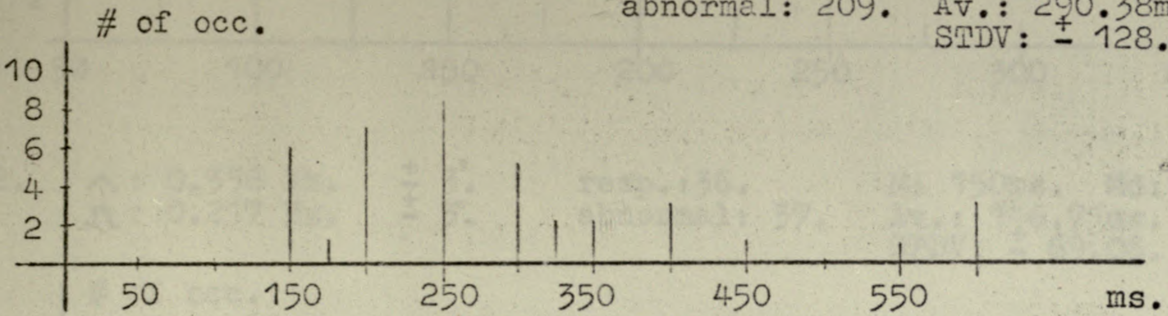
5. 1.230, 1.480, 1.710 Hz. resp.: 32. M: 225ms. Md: 250ms. Av.: 211.67ms. STDV: ± 58.99



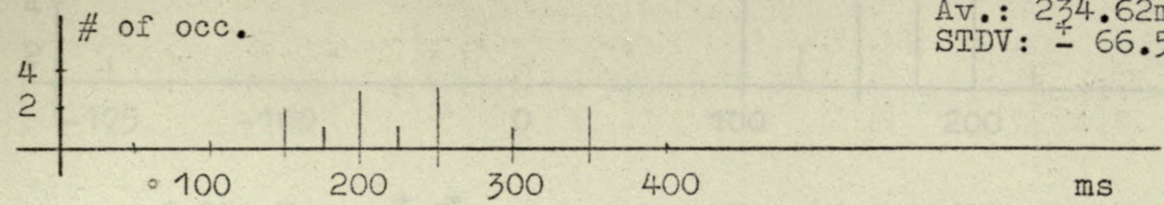
6. 1.480, 1.710, 1.950 Hz. resp.: 60. abnormal: 91. M: 225ms. Md: 250ms. Av.: 214.17ms. STDV: ± 61.47.



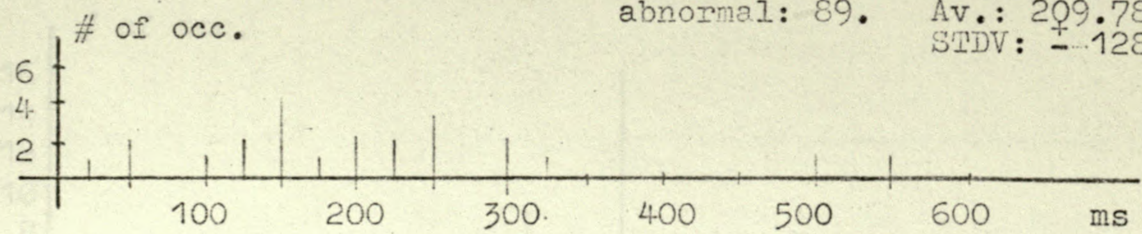
7. 1.710, 1.950, 2.210 Hz. resp.: 41. abnormal: 209. M: 250ms. Md: 250ms. Av.: 290.38ms. STDV: ± 128.58.



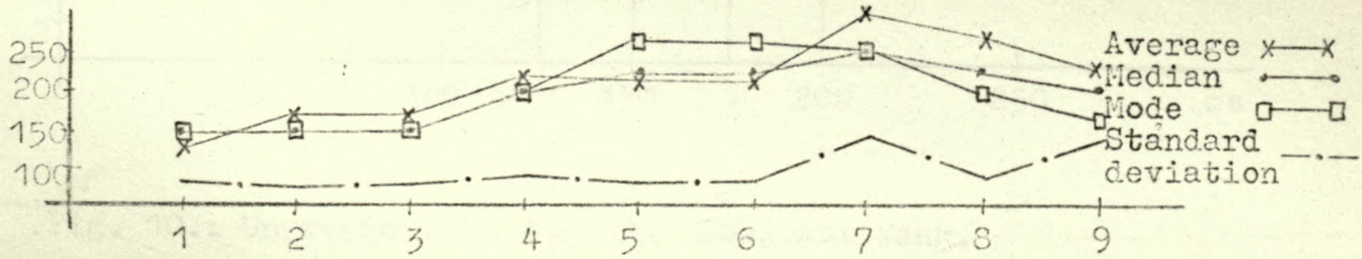
8. 1.950, 2.210, 2.450 Hz. resp.: 13. abnormal: 58. M: 225ms. Md: 200, 250ms. Av.: 234.62ms. STDV: ± 66.57.



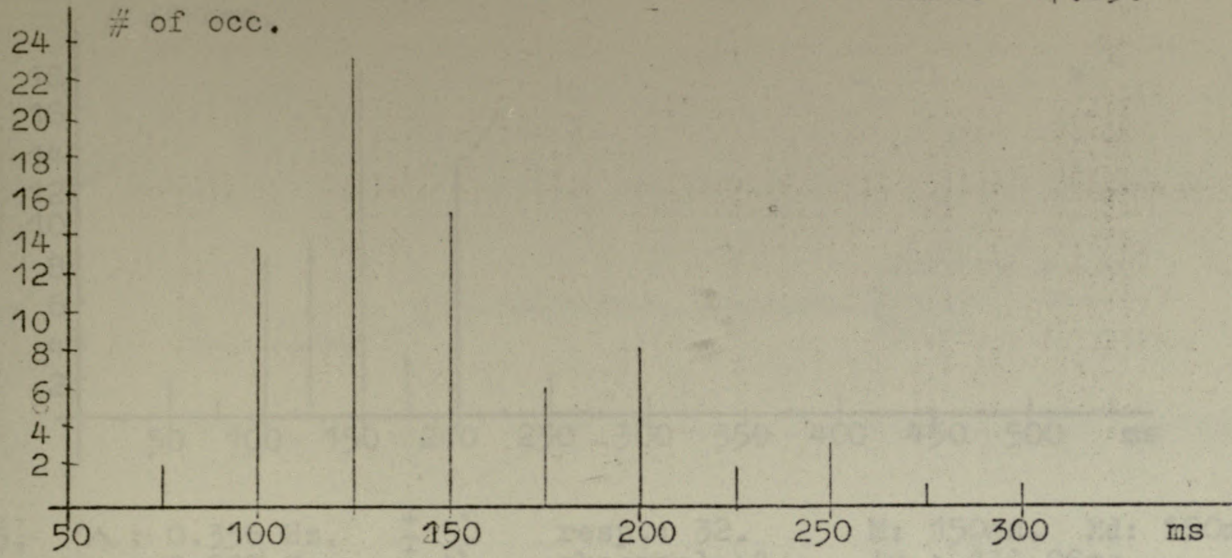
9. 2.210, 2.450, 2.730 Hz. resp.: 23. abnormal: 89. M: 200ms. Md: 150ms. Av.: 209.78ms. STDV: ± 128.31



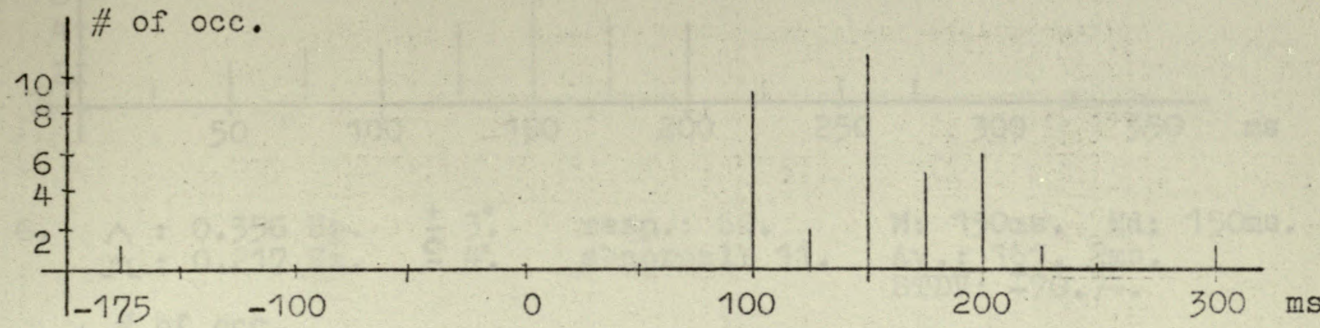
M vs. freq.



1. ∧ : 0.356 Hz. ± 3°. resp.: 74. M: 125ms. Md: 125ms.
⎍ : 0.217 Hz. ± 2°. abnormal: 34. Av.: 148.61ms.
STDV: ±47.63.



2. ∧ : 0.356 Hz. ± 3°. resp.:36. M: 150ms. Md: 150ms.
⎍ : 0.217 Hz. ± 3°. abnormal: 37. Av.: 146.71ms.
STDV: ± 69.06.



3. ∧ : 0.356 Hz. ± 3°. resp.: 43. M: 150ms. Md: 150ms.
⎍ : 0.384 Hz. ± 3°. abnormal: 30. Av.: 156.97ms.
STDV: ± 32.42.

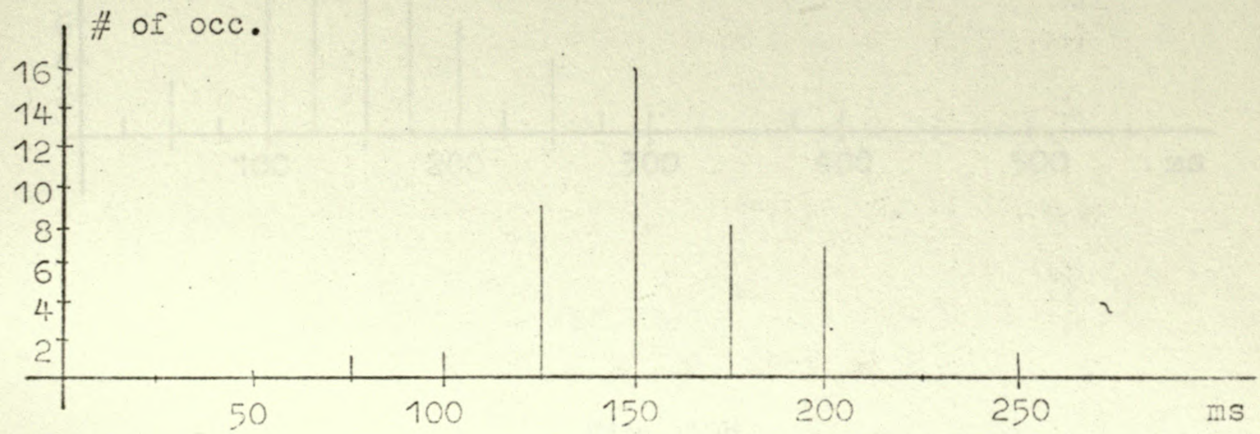
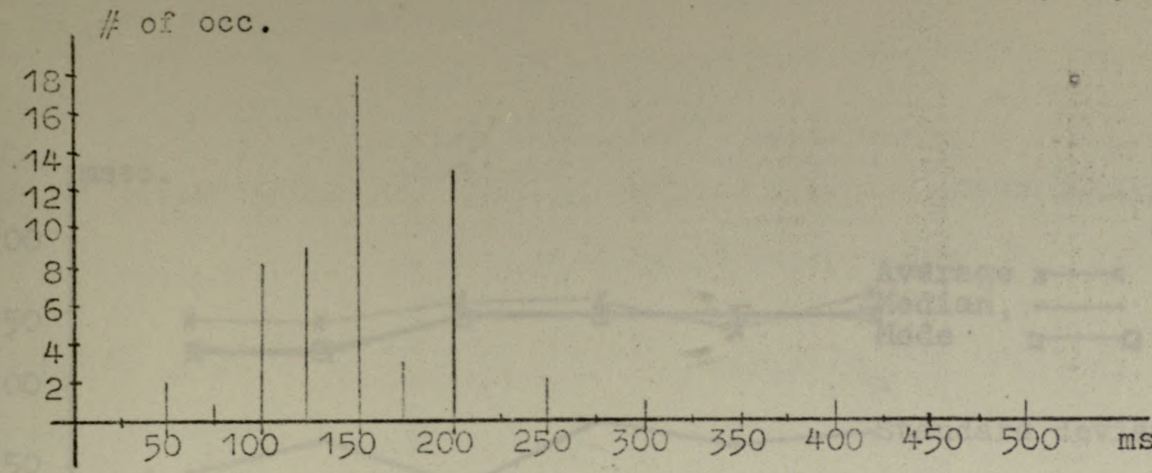
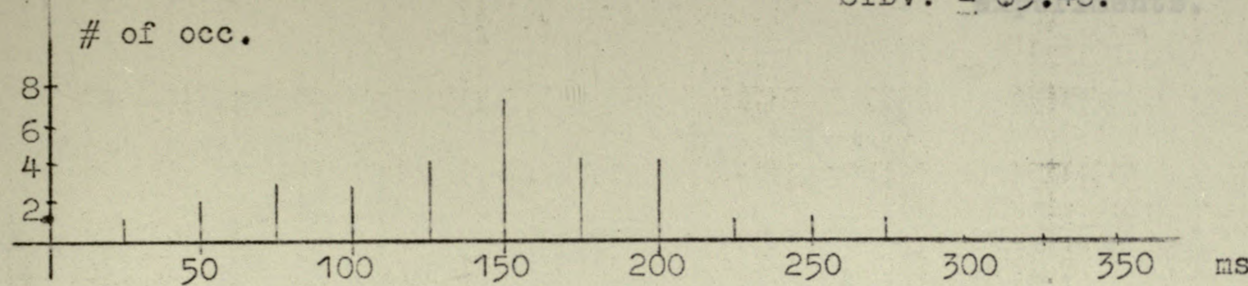


Fig. 10.: Unpredictable signal. Subject: Wang.

4. ∧ : 0.356 Hz. ± 4°. resp.: 61. M: 150ms. Md: 150ms.
⎍ : 0.217 Hz. ± 4°. abnormal: 26. Av.: 157.79ms.
STDV: ± 76.85.



5. ∧ : 0.356 Hz. ± 3°. resp.: 32. M: 150ms. Md: 150ms.
⎍ : 0.217 Hz. ± 4°. abnormal: 4. Av.: 139.06ms.
STDV: ± 63.48.



6. ∧ : 0.356 Hz. ± 3°. resp.: 60. M: 150ms. Md: 150ms.
⎍ : 0.217 Hz. ± 4°. abnormal: 11. Av.: 161. 2ms.
STDV: ±70.74.

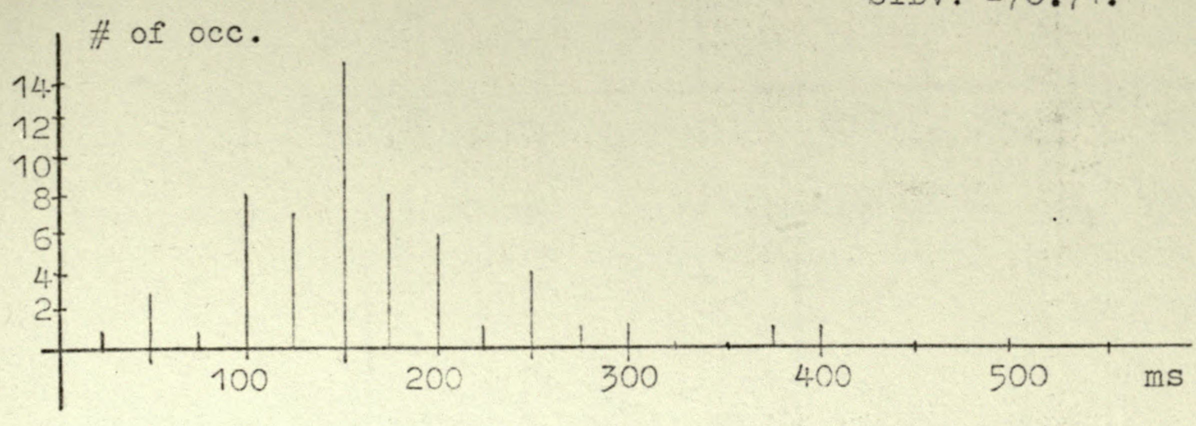


Fig. 10b.

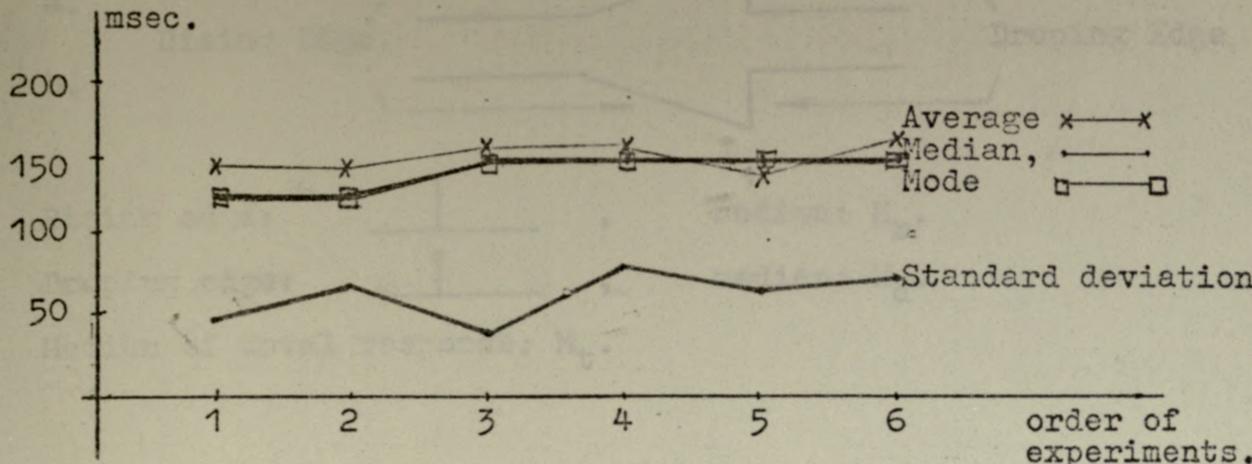
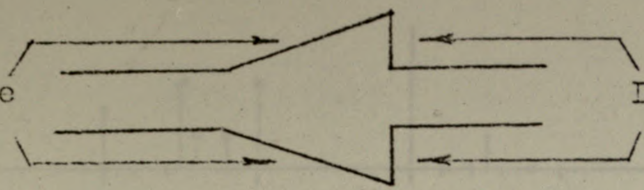
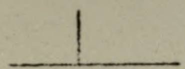

msec.
200
150
100
50
Average
Median,
Mode
Standard deviation
1
2
3
4
5
6
order of
experiments.

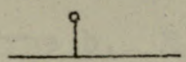
Fig. 10c.

Two kinds of triangular wave:

a. Rising Edge  Droping Edge

b.

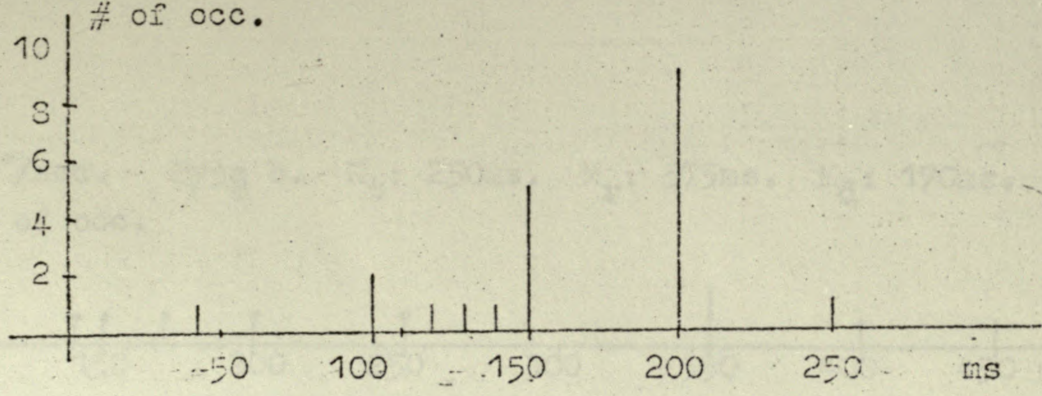
Rising edge:  , median: M_r.

Droping edge:  , median: M_d.

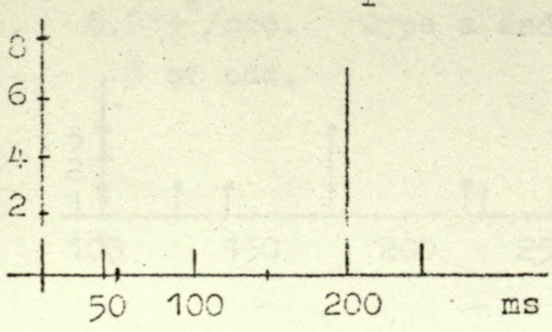
Median of total response: M_t.

1. 2°/sec. Type a.

Total response: M_t: 150ms.



Rising edge: M_r: 200ms.



Droping edge: M_d: 150ms.

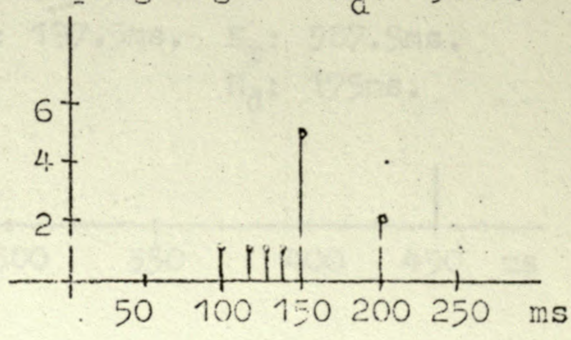
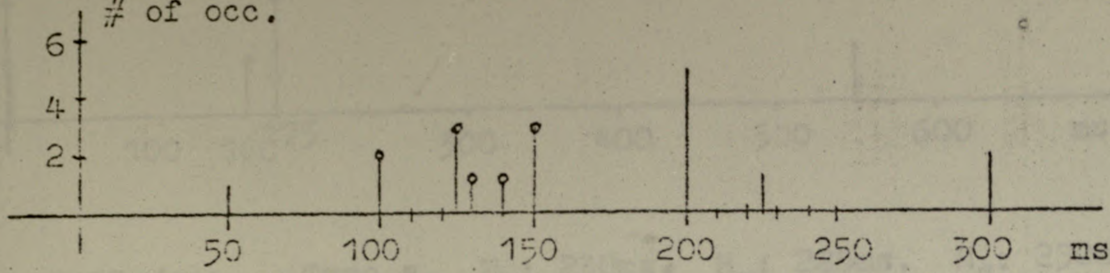
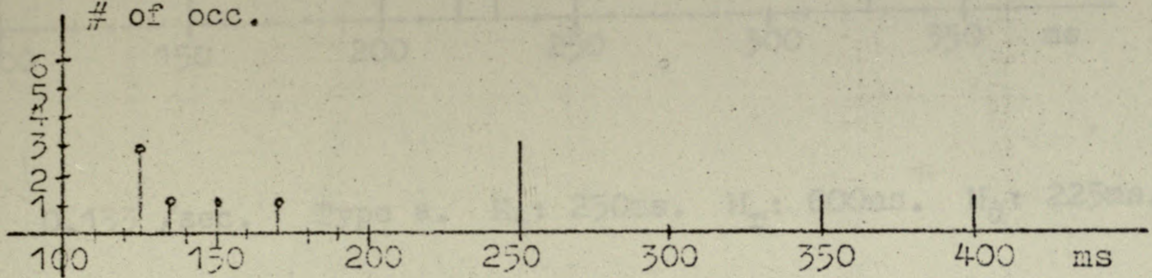


Fig. 11: Ramp function. Subject: Wang.

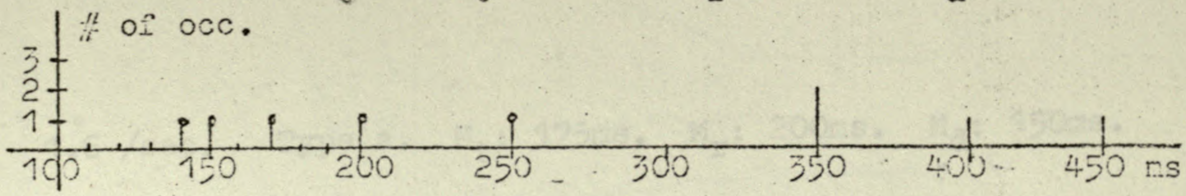
2. 2°/sec. Type b. M_t: 150ms. M_r: 250ms. M_d: 130ms.



3. 1°/sec. Type b. M_t: 170ms. M_r: 250ms. M_d: 130ms.



4. 0.7°/sec. Type b. M_t: 250ms. M_r: 375ms. M_d: 170ms.



5. 0.833°/sec. Type a and b. M_t: 197.5ms. M_r: 387.5ms.
M_d: 175ms.

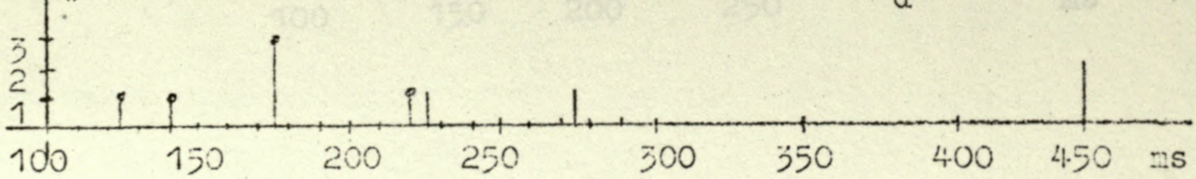
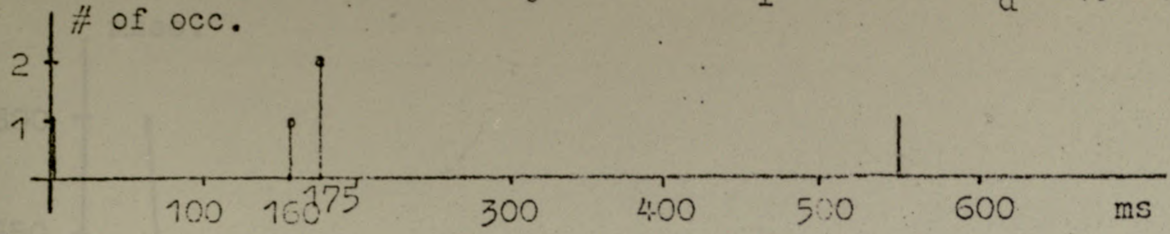
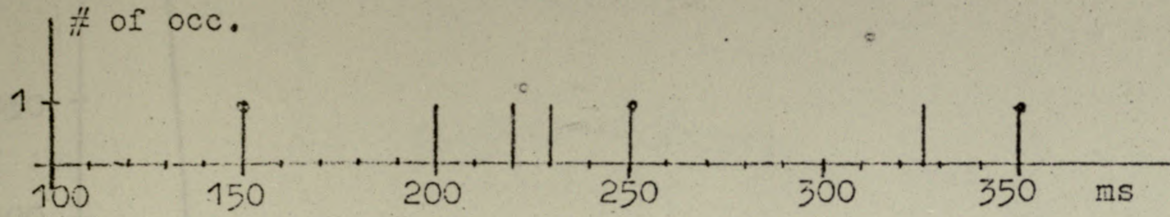


Fig. 11b.

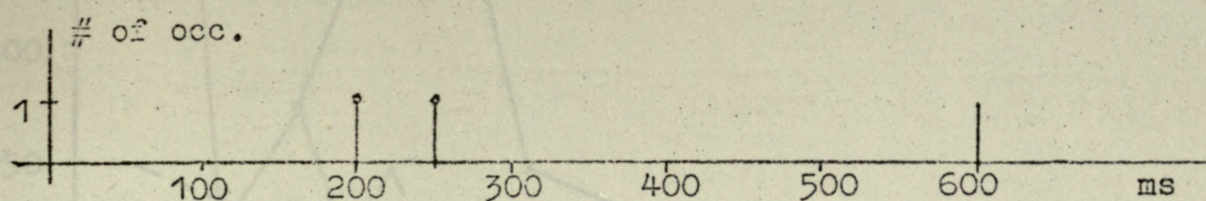
6. 0.5 /sec. Type a. M_t: 175ms. M_r: 275ms. M_d: 175ms.



7. 0.33 /sec. Type a. M_t: 230ms. M_r: 225ms. M_d: 250ms.



8. 0.133 /sec. Type a. M_t: 250ms. M_r: 600ms. M_d: 225ms.



9. 1.6 /sec. Type a. M_t: 175ms. M_r: 200ms. M_d: 150ms.

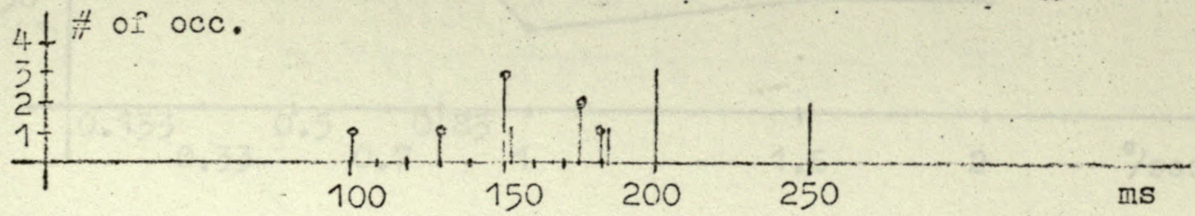


Fig. 11c.

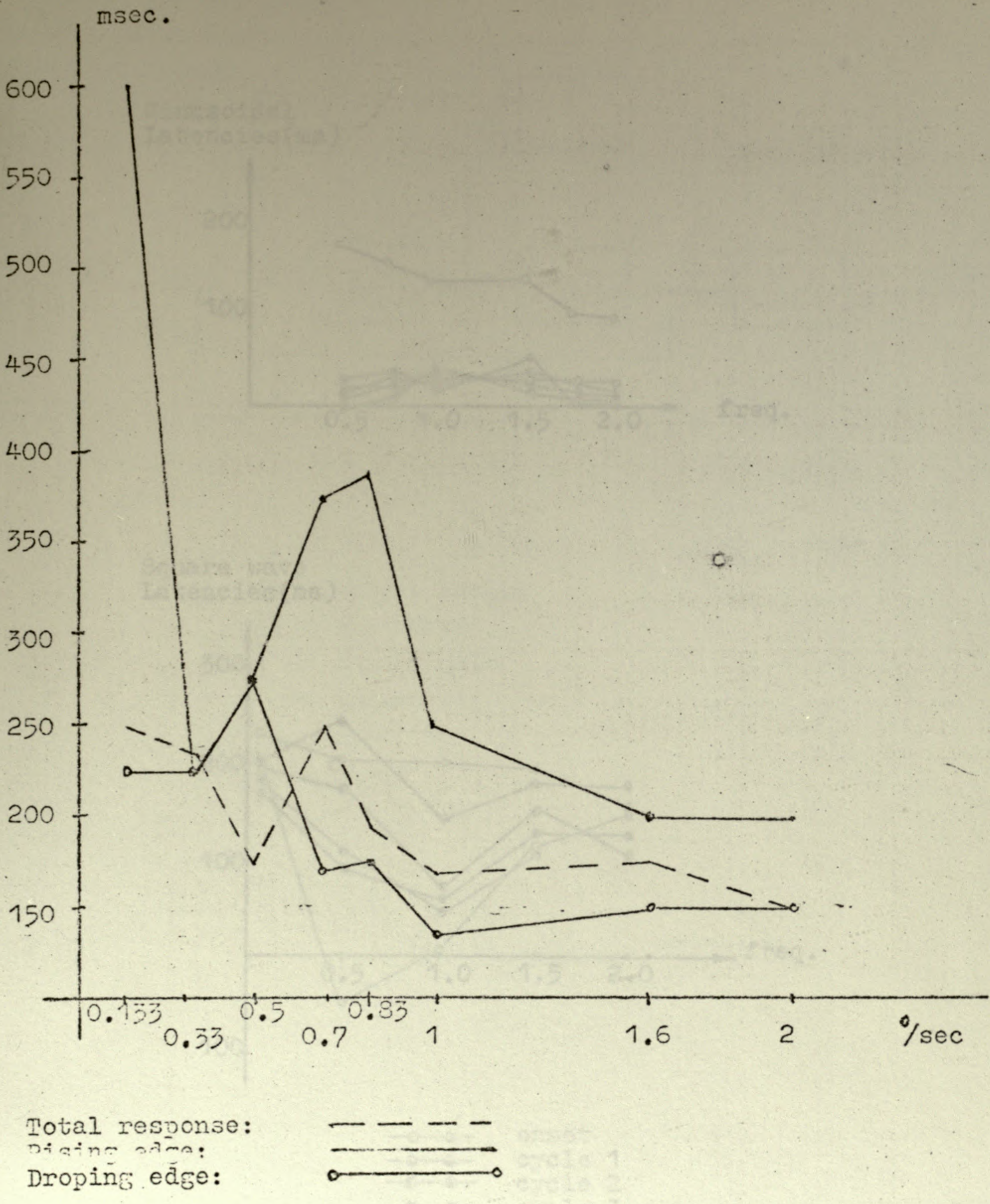

msec.
600
550
500
450
400
350
300
250
200
150
0.133
0.33
0.5
0.7
0.83
1
1.6
2
°/sec
Total response:
Rising edge:
Droping edge:

Fig. 11d.

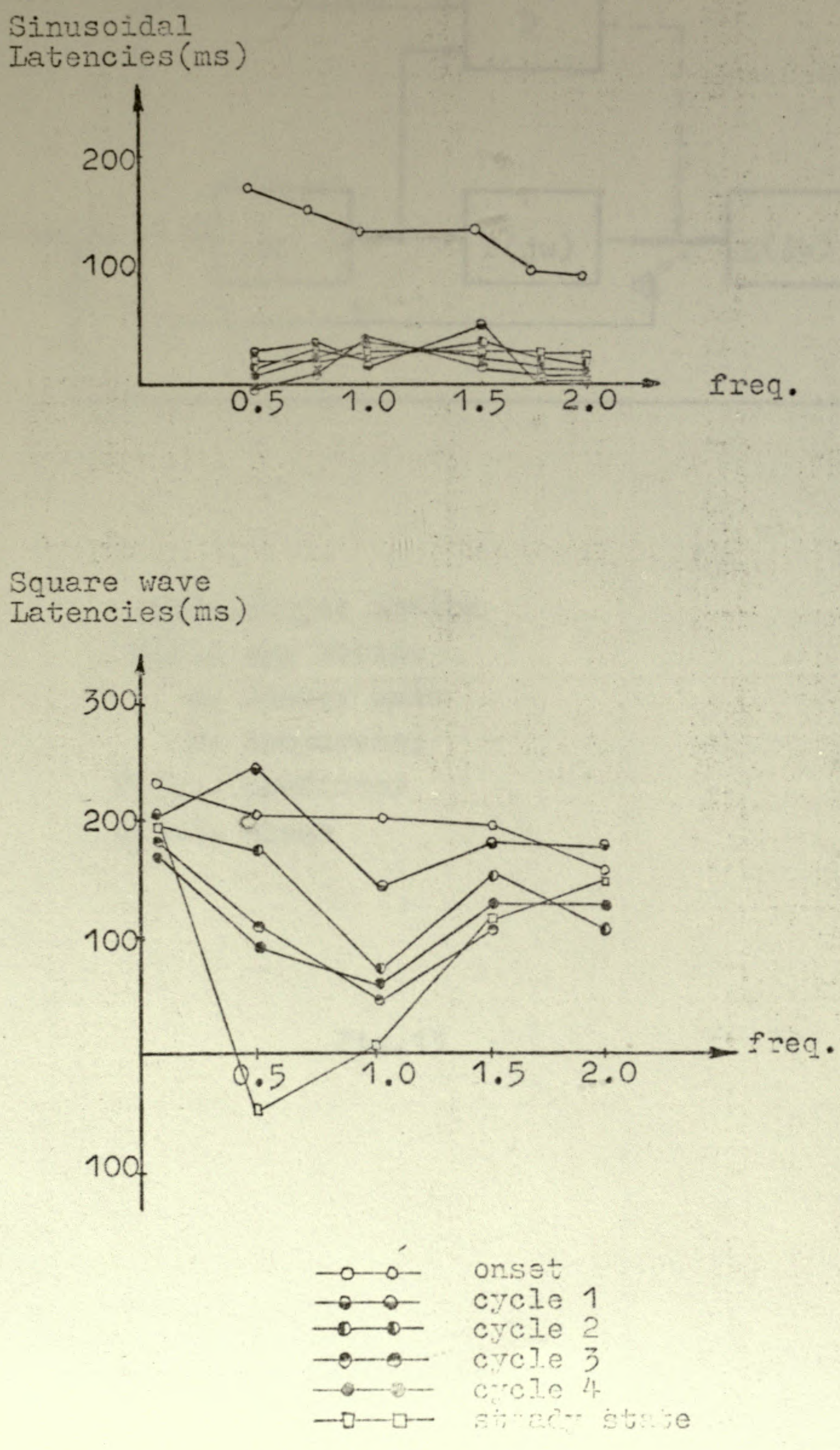
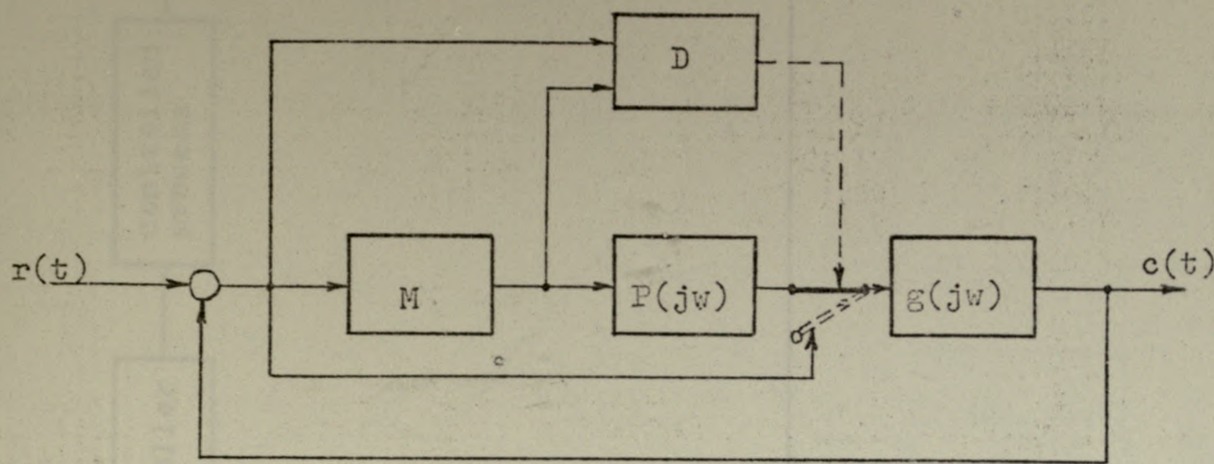

Sinusoidal
Latencies(ms)
200
100
0.5
1.0
1.5
2.0
freq.
Square wave
Latencies(ms)
300
200
100
0.5
1.0
1.5
2.0
freq.
100
onset
cycle 1
cycle 2
cycle 3
cycle 4
steady state

Fig. 12



r(t)= target motion
c(t)= eye motion
M= memory unit
D= comparator
P(jw)= predictor
g(jw)= plant

Fig.13

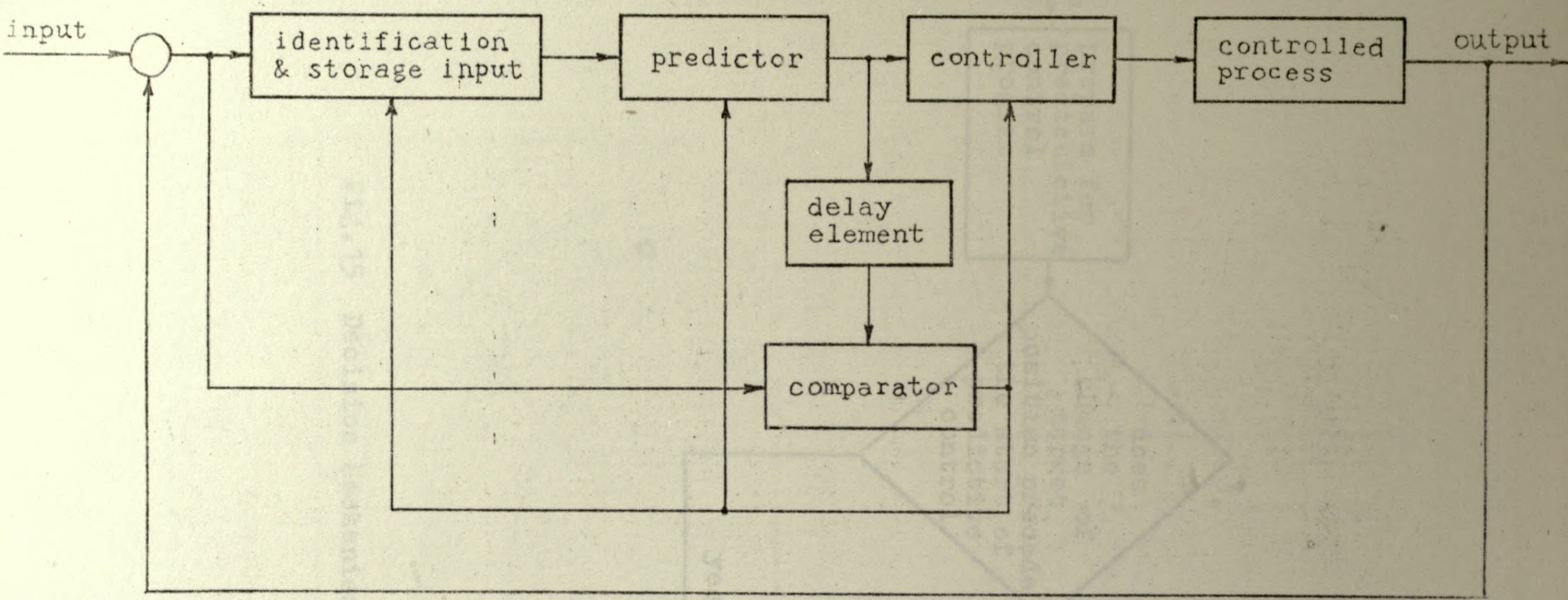

input
identification
& storage input
predictor
controller
controlled
process
output
delay
element
comparator

Fig. 14

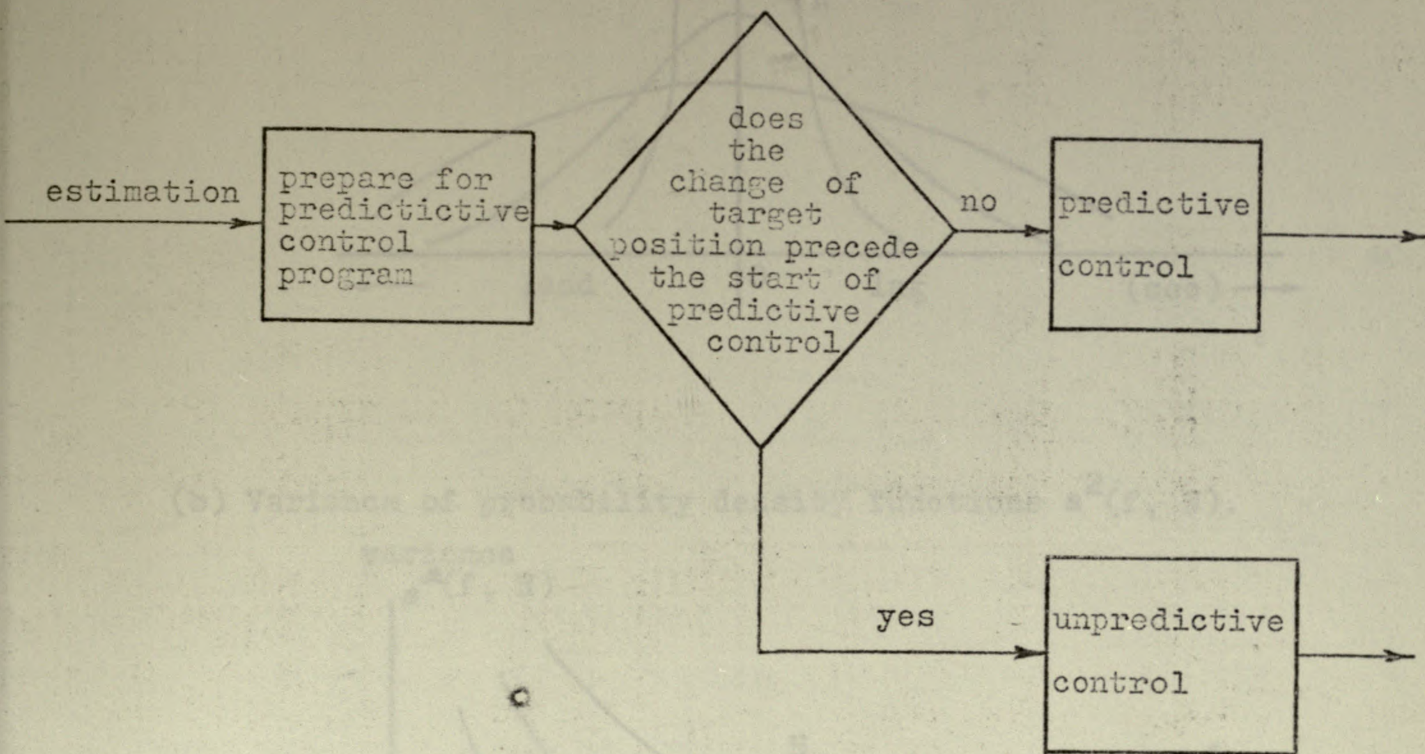
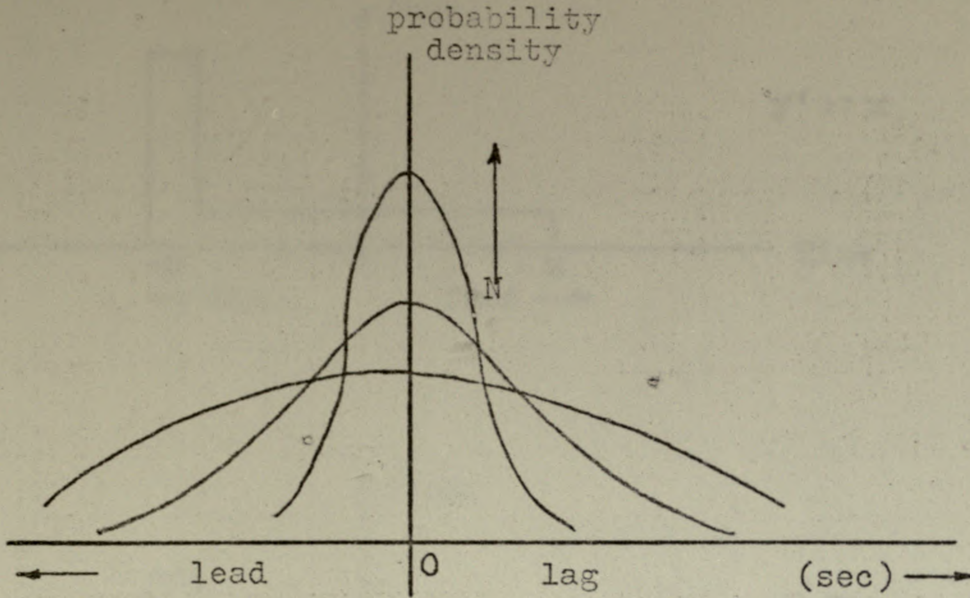


Fig.15 Decision Mechanism

(a) Probability density functions of estimated input period.



(b) Variance of probability density functions $s^2(f, N)$.

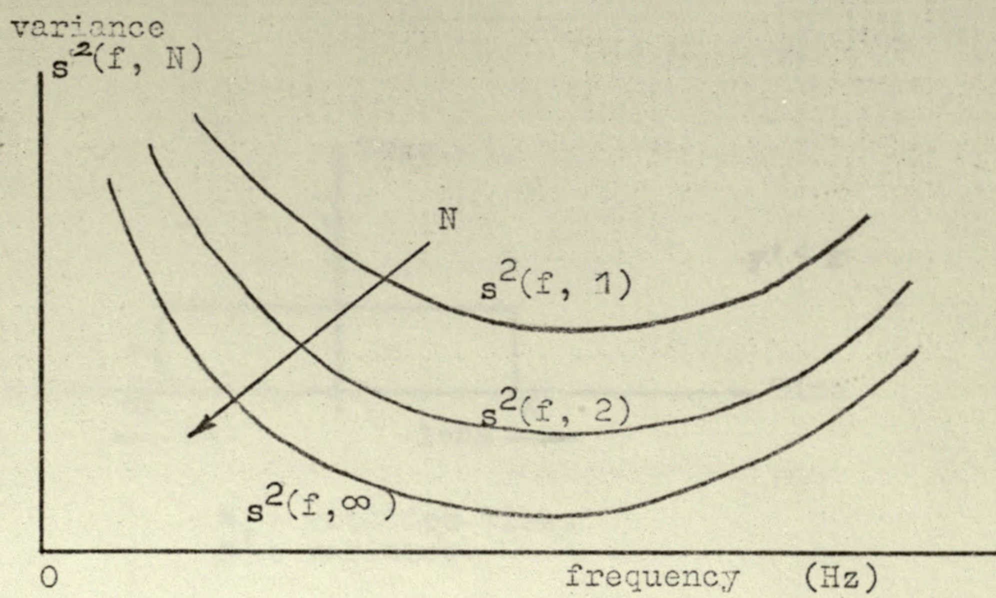
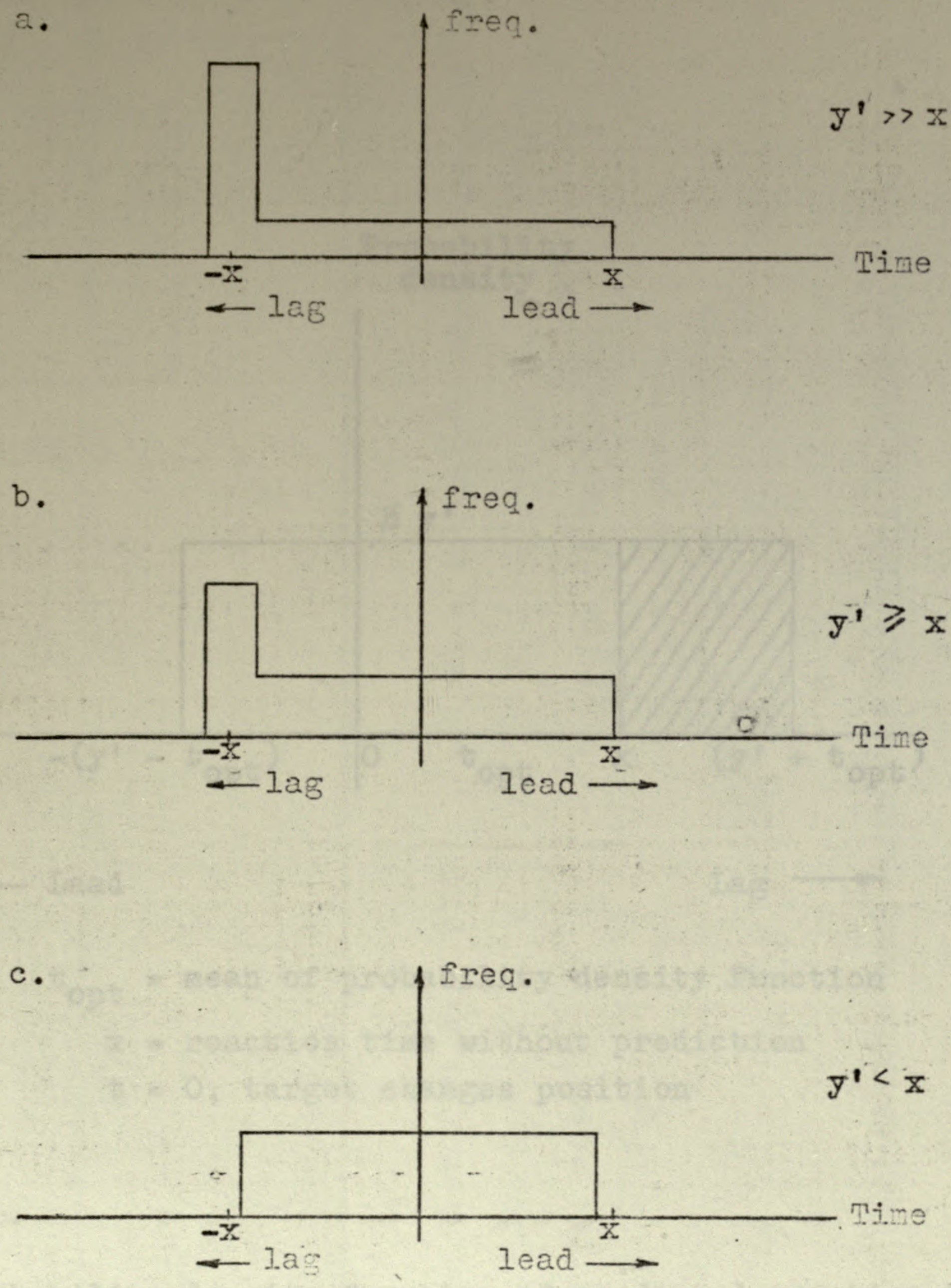
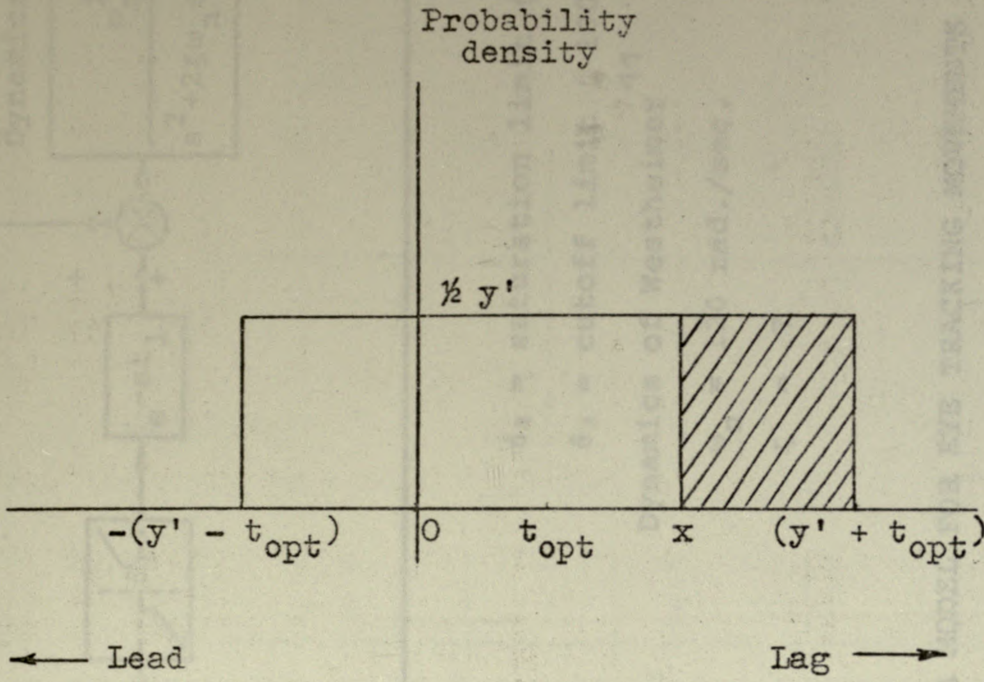


Fig. 16



x : reaction time.
y' : variance.

Fig.17



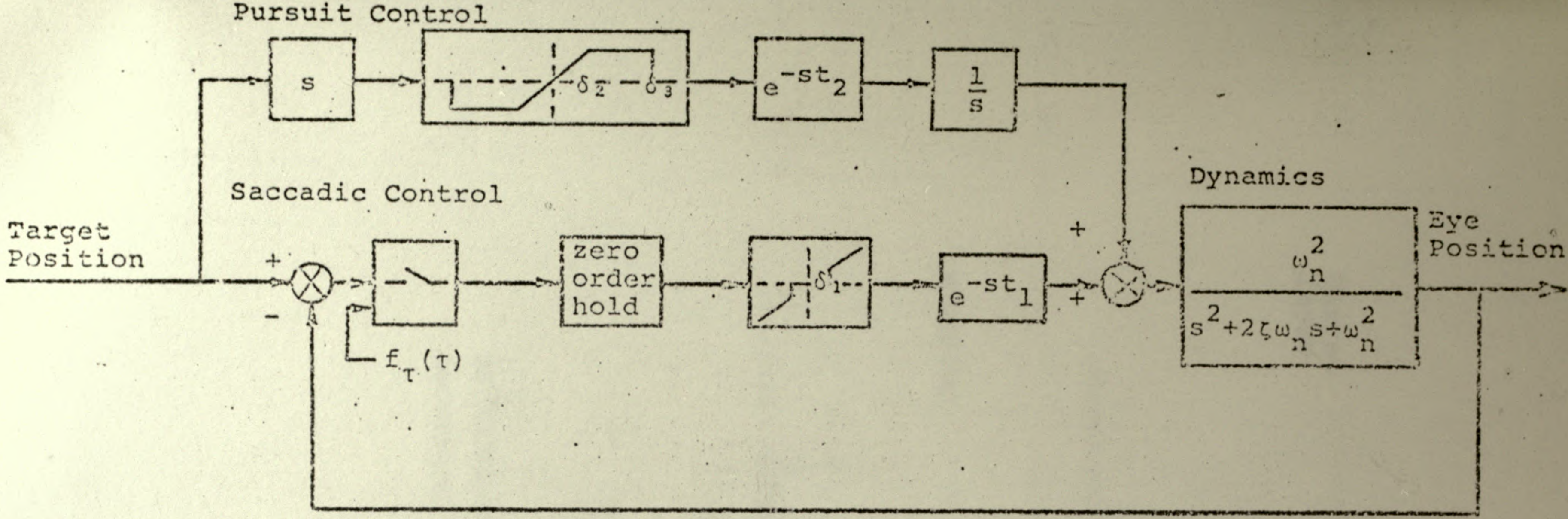
$\bar{t}_{opt}$ = mean of probability density function

x = reaction time without prediction

when $t = 0$, target changes position

Probability density function of predicted reaction time.

Fig. 18



$f_\tau(\tau)$ = intersample time distribution

t_1 = saccadic delay (100 msec.)

t_2 = pursuit delay (134 msec.)

δ_1 = foveal dead zone ($.3^\circ$)

δ_2 = saturation limit (25 - 30 $^\circ$/sec.)

δ_3 = cutoff limit (100 $^\circ$/sec.)

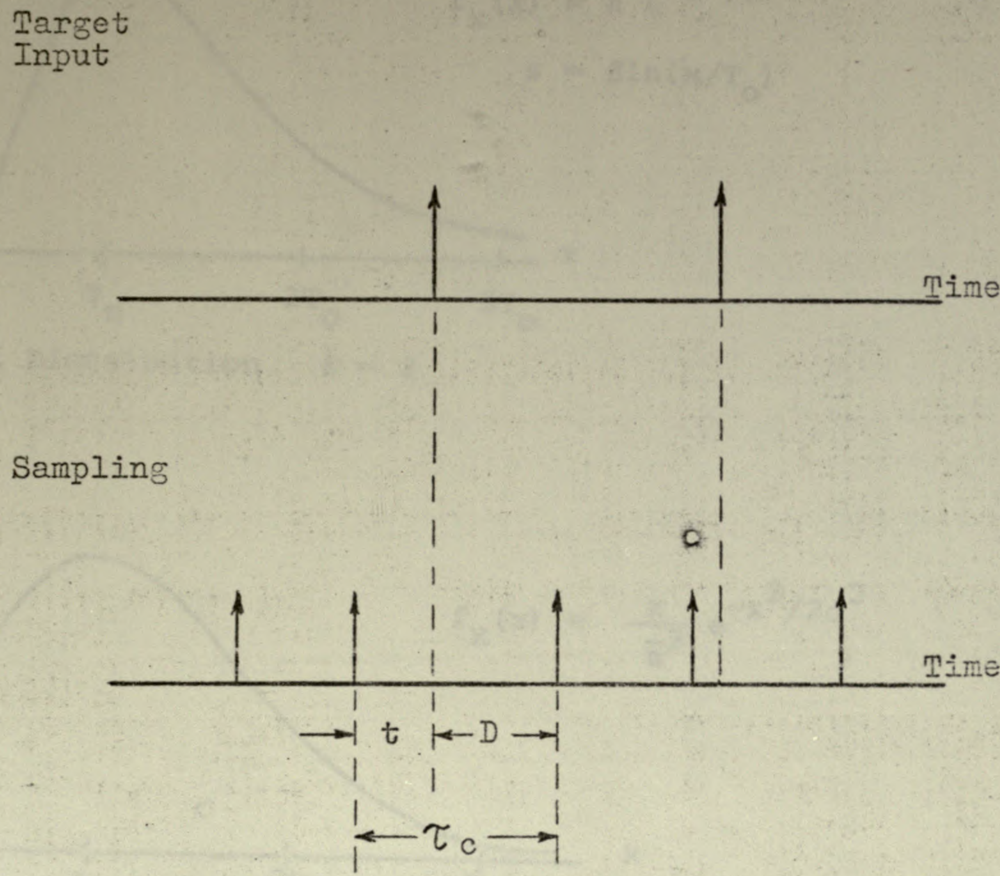
Dynamics of Westheimer[11]

ω_n = 120 rad./sec.

ζ = .7

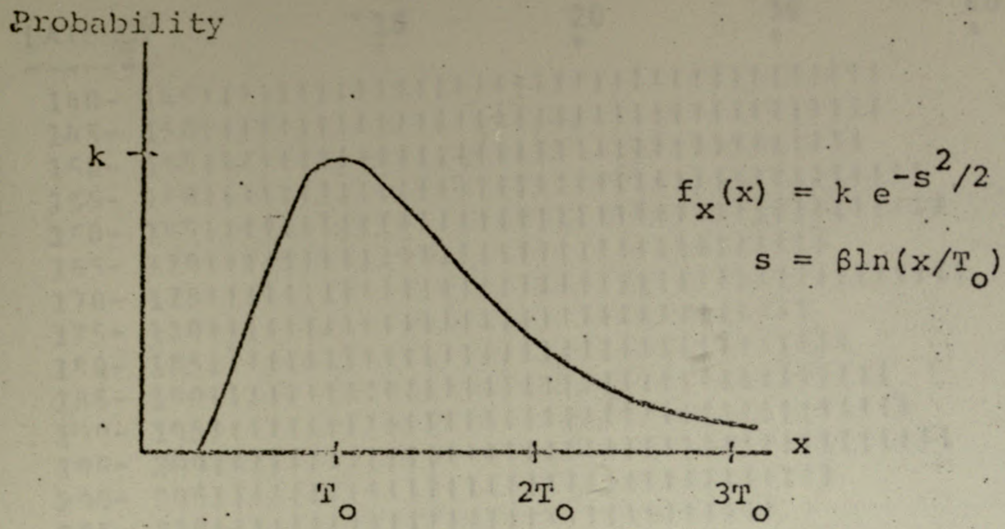
A REVISED SAMPLED DATA MODEL FOR EYE TRACKING MOVEMENTS

Fig. 19

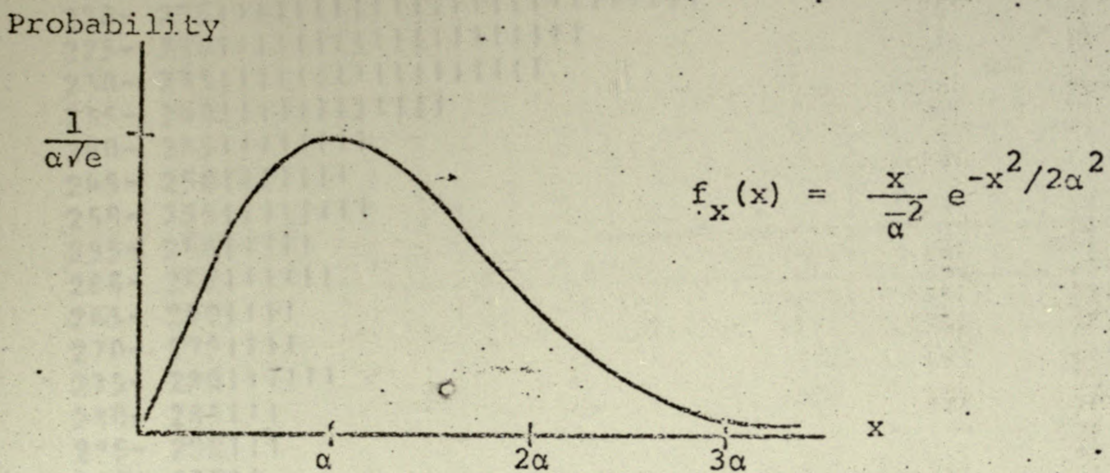


Definition of symbols showing inputs occurring in sampling intervals τ_c.

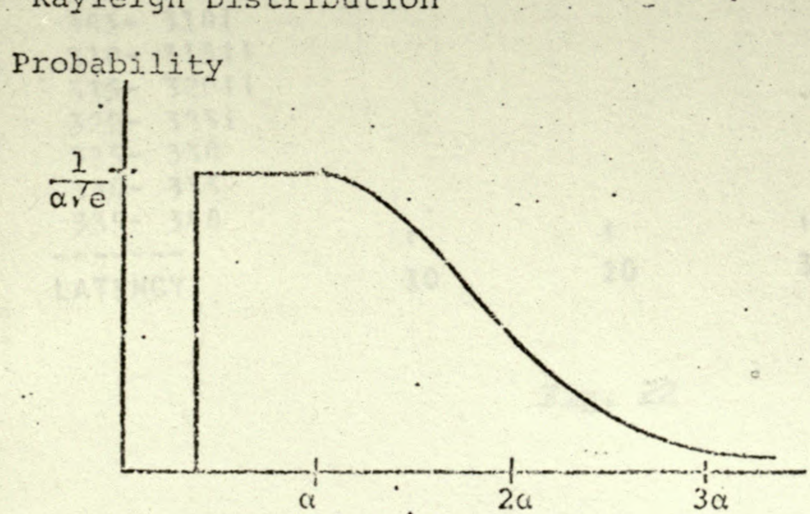
Fig. 20



a) Log Normal Distribution $\beta = 2$



b) Rayleigh Distribution



c) Non-increasing fit to Rayleigh Distribution

Mathematical Descriptions of Latency Histograms

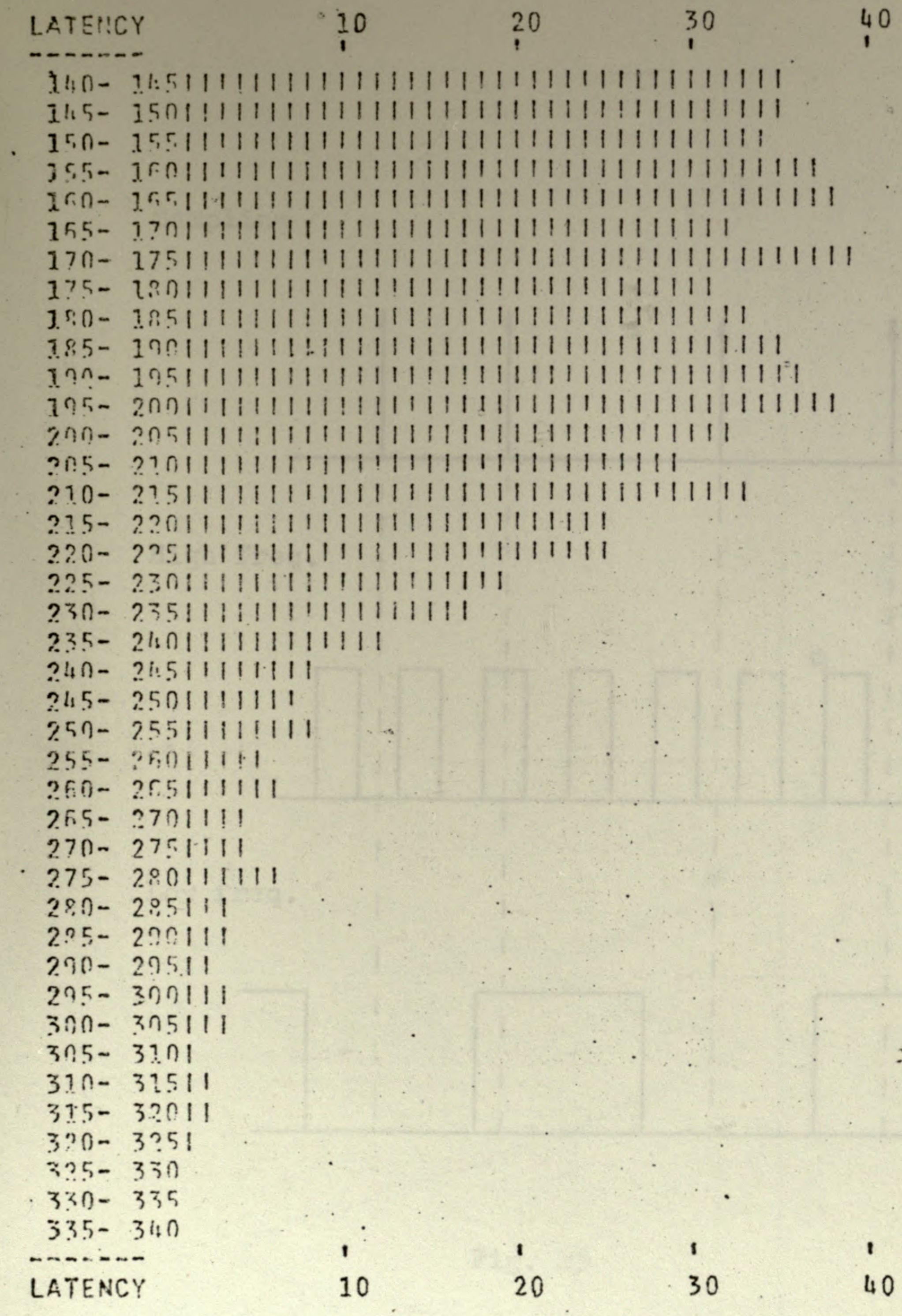

LATENCY
10
20
30
40
140- 145
145- 150
150- 155
155- 160
160- 165
165- 170
170- 175
175- 180
180- 185
185- 190
190- 195
195- 200
200- 205
205- 210
210- 215
215- 220
220- 225
225- 230
230- 235
235- 240
240- 245
245- 250
250- 255
255- 260
260- 265
265- 270
270- 275
275- 280
280- 285
285- 290
290- 295
295- 300
300- 305
305- 310
310- 315
315- 320
320- 325
325- 330
330- 335
335- 340
LATENCY
10
20
30
40

Fig. 22

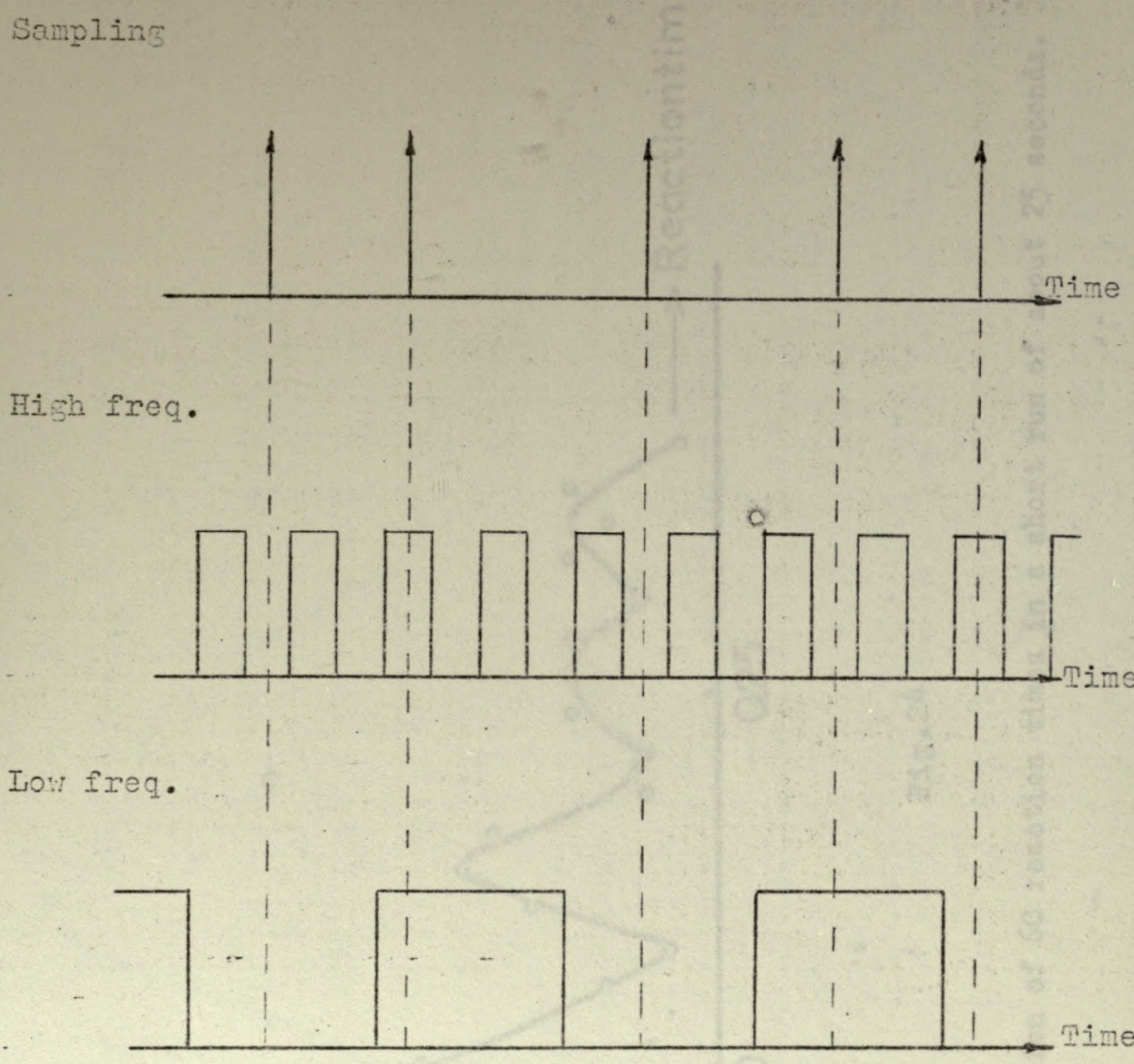


Fig. 23

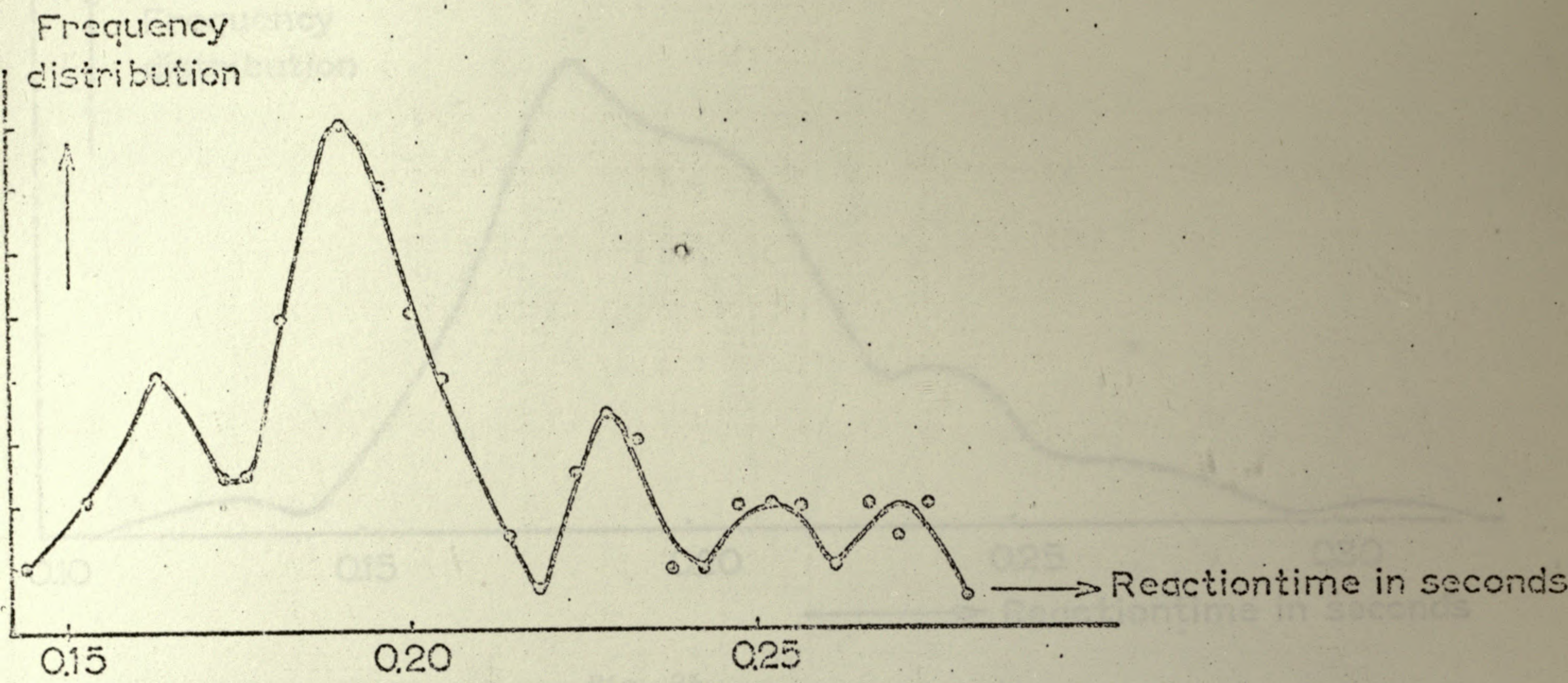


Fig.24

The frequency distribution of 60 reaction times in a short run of about 25 seconds.

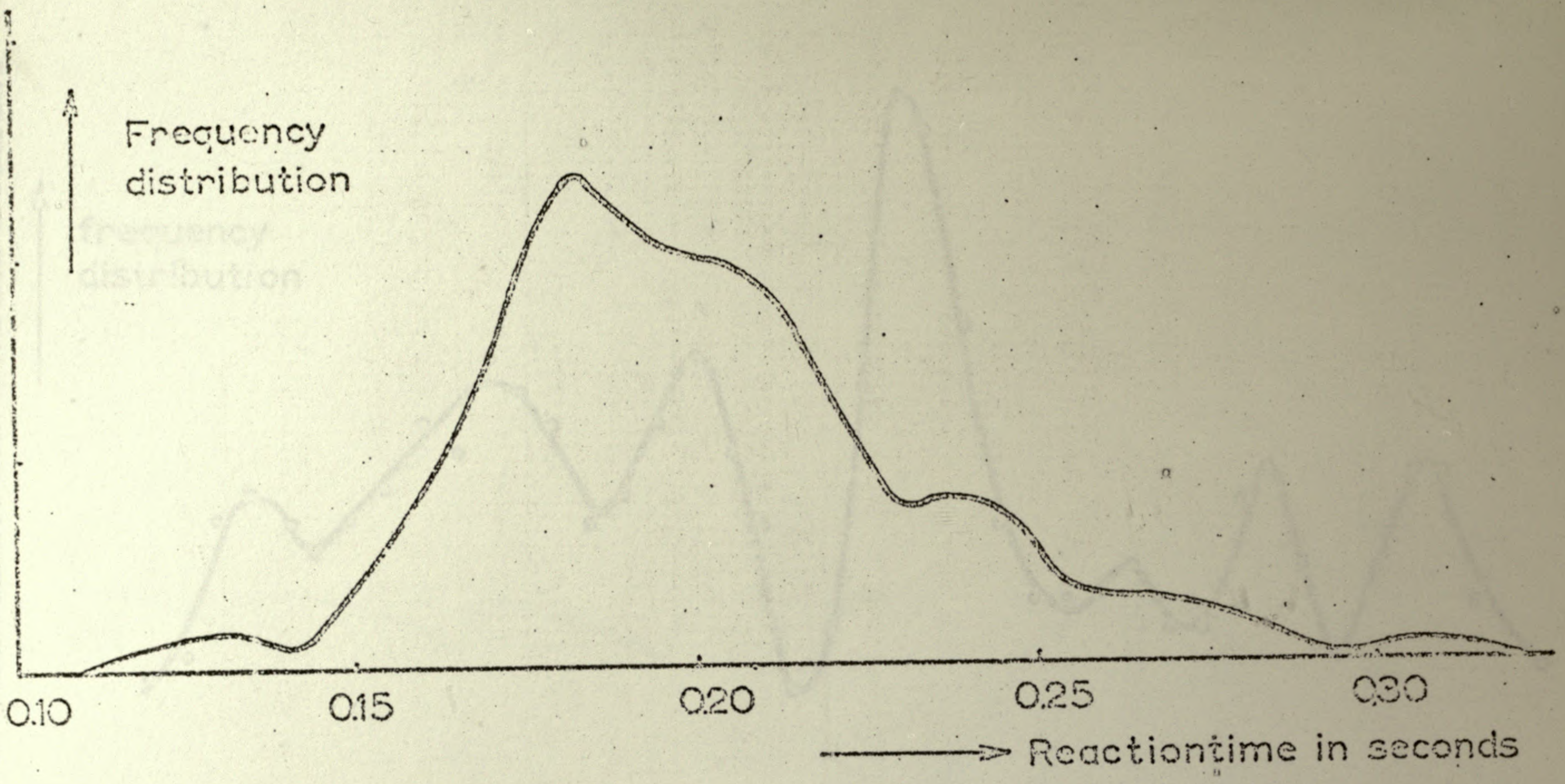


Fig. 25

The frequency distribution of 600 reaction times in a long run lasting about 30 minutes.

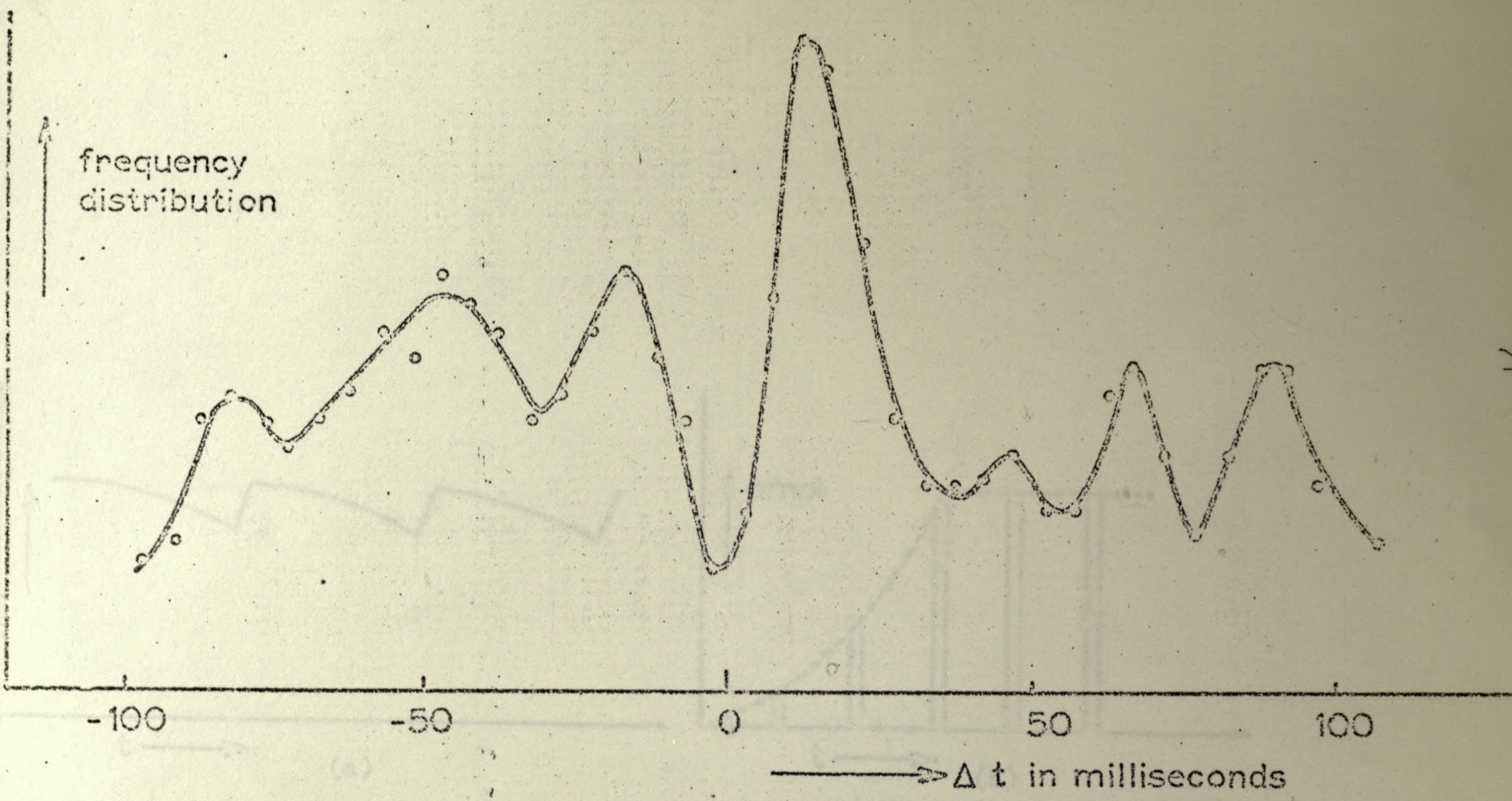


Fig. 26

The frequency distribution of the difference of successive reaction times from Fig. 25

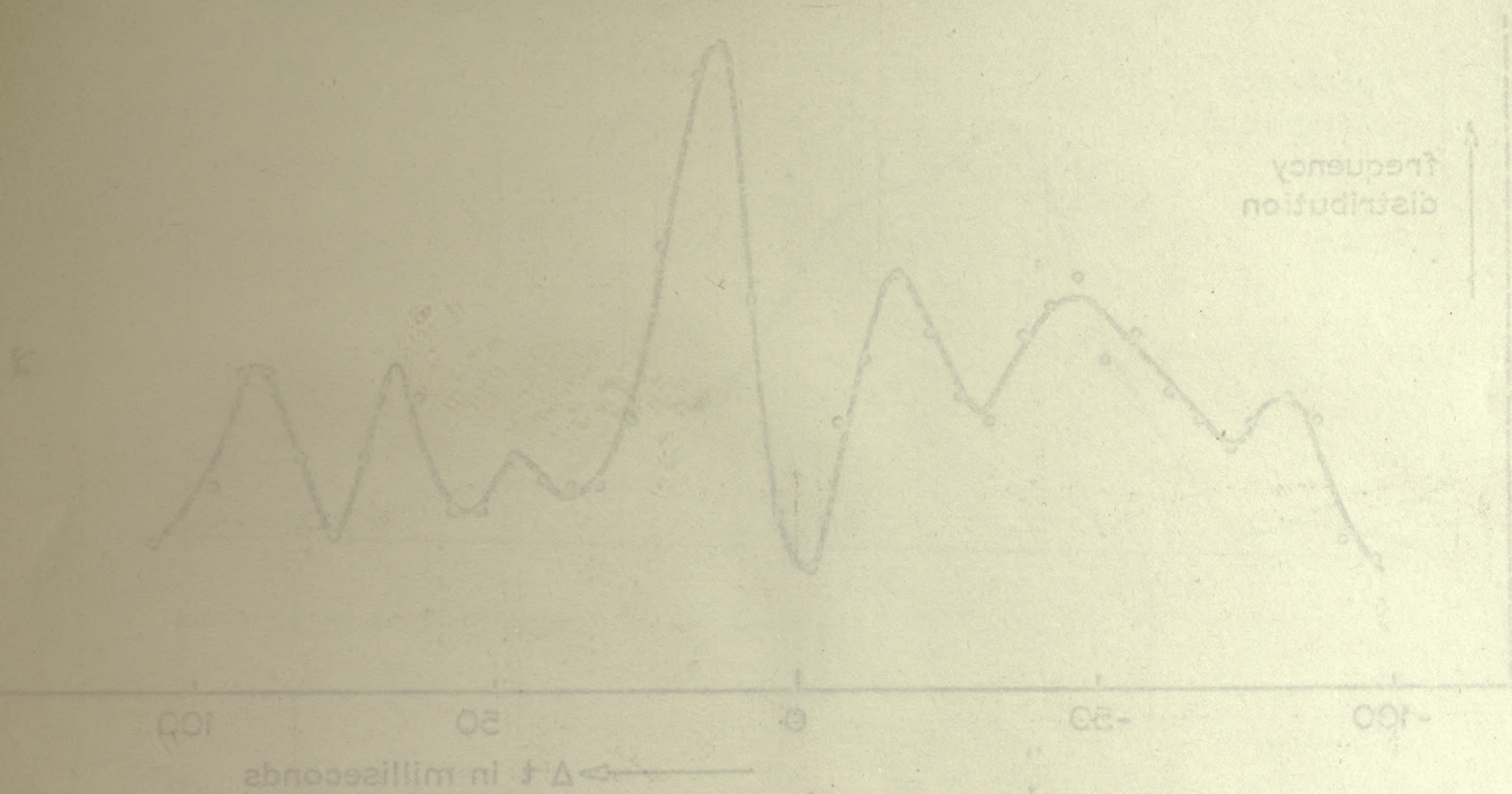


Fig. 26

The frequency distribution of the difference of successive reaction times from Fig. 25

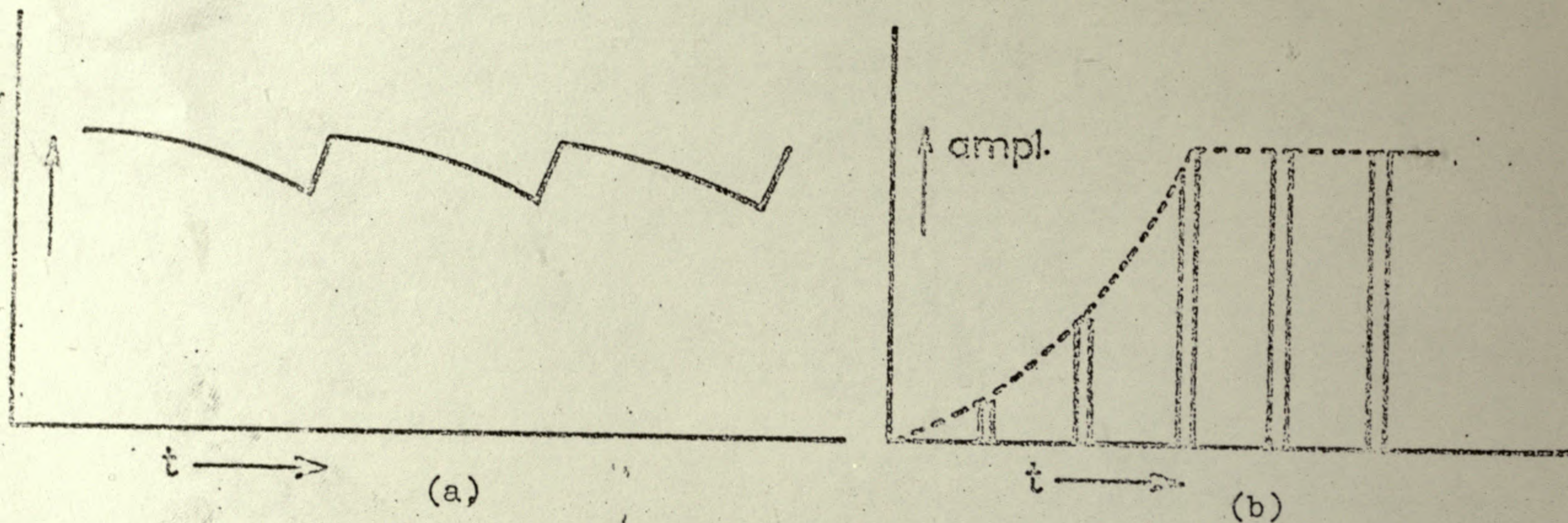


Fig. 27

The sawtooth representation of $\overline{t}= l_1+l_2$.

The amplitude of the signal at the output of the oscillating amplifier as a function of time.